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ESTABLISHMENT OF RAINFALL INTENSITY-DURATION-FREQUENCY CURVES OF KHULNA

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ABSTRACT

The Intensity-Duration-Frequency (IDF) is one of the most important hydrologic tools used by engineers for designing drainage and flood control structures in urban areas. Local IDF equations are often estimated on the basis of records of intensities abstracted from daily rainfall depths of different durations, observed at a given recording rainfall gauging station. Because in developing countries like Bangladesh short duration rainfall is scarce and only daily rainfall data is available. In such cases, design rainfall approximation isn't accurate which can lead to failure of drainage system. Khulna is an important city of Bangladesh which is situated at the southern part of the country. It is the third-largest city in Bangladesh. It is the administrative seat of Khulna District and Khulna Division. Because of having the largest seaport of the country, it is an important hub for the industry and economy of Bangladesh. In this paper, the time scale invariance of rainfall of Khulna are investigated and Intensity-Duration-Frequency (IDF) curves for this city is determined. For this process, three empirical equations are used and the results are compared to find out the best fitted rainfall intensity method for the study area. This method allows for the determination of the design value of rainfall of selected return period and durations shorter than a day by using only the daily data. Firstly, 62 years of rainfall data (1948-2010) are collected from BMD and the quality of data is analyzed. The homogeneity, consistancy, randomness of data are checked and fitted to Gumbel Type 1 Extreme Value distribution function. The maximum rainfall intensities for 2 year,5 year, 10 year, 25 year, 50 year and 100 year return period were calculated using Gumbel distribution. Design rainfall from those analyses are used to calculate short duration rainfall intensity by using three different equations : Talbot, Sherman and Kimjima equation. Among these three empirical formulas, least-square method is applied to determine the best fitted rainfall intensity method for the study area. From the RMSE values of these three equations, it is concluded that Sherman equation is best fitted for this region. Finally, a 2hour-5year hyetograph is generated for Khulna region using the Sherman formula.

Keywords: *IDF curve, Gumble distribution, Khulna, RMSE value, Short duration data.*

1. INTRODUCTION

The Intensity-Duration-Frequency (IDF) is one of the most commonly used tools used by water resources engineers for planning, designing and operating drainage and flood control structures in urban areas. The establishment of such relationships was done before by Sherman (1905) and Bernard (1932). Since then, many relationships have been constructed for several parts of the globe. Hershfiled (1961) developed various rainfall contour maps to provide the design rain depths for various return periods and durations. Bell (1969) and Chen (1983) derived the IDF formulae for the United States, Kouthyari & Garde (1992) presented a relationship between rainfall intensity and duration for India.

Local IDF equations are generated from the records of intensities abstracted from rainfall depths of different durations, observed at a given recording rainfall gauging station. But only in some regions, these rainfall gauging stations are operating for a long time which produce a good amount and quality of data in order to produce reliable IDF relationship for the area. But mostly in developing countries, these rainfall gauging station doesn't exist or the sample size is too small to generate the IDF curve for the region. Specially in south asian subcontinent region and in Bangladesh, this problem is quite prominent. In this regard, research has focused on the mathematical representation of rainfall both in time and space, in which Gupta & Waymire (1990), Burlando & Russo (1996), Menabde et al. (1999), De Michele et al. (2002), Pao-Shan-Yu et al. (2004) and Nhat et al. (2006) showed the scaling invariance models to derive IDF characteristics of short duration rainfall from daily data.

Earlier, in Bangladesh different IDF curves were developed for different parts of Bangladesh. Matin et al. (1984) developed IDF curve for North-East region of Bangladesh. Recently, Rasel et al. (2015) developed IDF for for North-West region Bangladesh, Chowdhury et al. (2007), Rashid et al. (2012) developed IDF for Sylhet city, Afrin et al. (2015) developed IDF for Dhaka city. Khulna is also an important divisional city of Bangladesh shown in Figure 1. This paper aims to analyse the time scale invariance of rainfall of Khulna and develop Intensity-Duration-Frequency (IDF) relationships for this city.

Figure 1 : Map of Study Area (Khulna)

2. METHODOLOGY

For this study, 62 years of rainfall data (1948-2010) are collected from BMD for Khulna city and the quality of data is analyzed. The homogeneity, consistency, randomness of data is checked and fitted to Gumbel Type 1 Extreme Value Distribution function. The maximum rainfall intensities for 2 years, 5 years, 10 years, 25 years, 50 years and 100 year return period were calculated using Gumbel distribution. Design rainfall from those analyses are used to calculate rainfall intensity by using three different equations: Talbot, Sherman and Kimjima equation. Among these three empirical formulas, least-square method is applied to determine the best fitted rainfall intensity method for the study area. From the RMSE values of these three equations, it is concluded that Sherman equation is best fitted for this region.

2.1 Generalized IDF relationship

According to Koutsoyiannis et al. (1998), the generalized IDF relationships are shown in equation (1) where i is the rainfall intensity of duration d, and w, ν, θ, and η are non-negative coefficients.

$$i = \frac{w}{(d^{\nu} + \theta)^{\eta}} \tag{1}$$

Koutsoyannis et al. (1998) also showed that the errors resulting from imposing $\nu=1$ in equation (1) are much smaller than the typical parameter and quantile estimation errors from limited size samples of data and considering $\nu \neq 1$ as a model over parameterization. Thus, Koutsoyannis et al. (1998) suggested for a given return period the general IDF relationships as

$$i = \frac{w}{(d + \theta)^{\eta}} \tag{2}$$

The coefficients w, θ, and η are not independent on the return period and this dependence cannot be arbitrary. The IDF curves for different return periods cannot intersect each other. This restriction, the range of variation of parameters w, θ, and η are limited. If $\{w_1, \theta_1, \eta_1\}$ and $\{w_2, \theta_2, \eta_2\}$ denote the parameter sets for return periods T_1 and T_2 respectively, with $T_2<T_1$, Koutsoyannis et al. (1998) suggest the following restrictions shown in equation (3).

$$\theta_1 = \theta_2 = \theta > 0;\ 0 < \eta_1 = \eta_2 = \eta < 1;\ w_1 > w_2 > 0 \tag{3}$$

In these restrictions, the only parameter that can consistently increase with increasing return periods is w and these arguments justify the formulation of the following general model for IDF relationship shown in equation (4).

$$i = \frac{a(T)}{b(d)} \tag{4}$$

which exhibits the great advantage of expressing separable relations between i and T, and between i and d. In equation (4), $b(d)=(d+\theta)^{\eta}$ with $\theta>0$ and $0<\eta<1$, whereas a(T) is completely defined by the probability distribution function of the maximum rainfall intensities.

3. RESULTS AND CONCLUSIONS

3.1 Gumble Type 1 Extreme Value Distribution

The data quality was good and normally distributed and Gumble Type 1 Extreme value distribution fits well in the rainfall intensities of 24 hours. The frequency analysis was done by Gumble's distribution using following equations:

$$X_T = u + \alpha y_T \tag{5}$$

$$u = \bar{x} - 0.5772\alpha \tag{6}$$

$$\alpha = S_x (\sqrt{6}/\pi) \tag{7}$$

$$y_T = - \ln(- \ln(1-1/T)) \tag{8}$$

The frequency analysis was done for 2 years, 5 years, 10 years, 20 years, 30 years, 50 years and 100 year return period. For example, Gumble distribution for 100 year return period is shown in table 1.

Table 1: Gumble distribution for 100 years return period for Khulna

Days	**Mean (mm)**	**Standard Deviation**	**α**	**u**	**T**	**y_T**	**X_T (mm)**	**Hours**	**Intensity (mm/hr)**
1D	134.54	70.72	55.14	102.72	100.00	4.60	356.37	24.00	14.85
2D	179.35	84.35	65.76	141.39	100.00	4.60	443.91	48.00	9.25
3D	205.85	98.42	76.74	161.55	100.00	4.60	514.55	72.00	7.15
4D	226.84	102.04	79.56	180.92	100.00	4.60	546.91	96.00	5.70
5D	243.95	103.56	80.74	197.34	100.00	4.60	568.78	120.00	4.74
6D	257.97	107.26	83.63	209.70	100.00	4.60	594.42	144.00	4.13
7D	273.85	109.76	85.58	224.46	100.00	4.60	618.14	164.00	3.77

3.2 Khulna IDF curve from long duration data

From the daily rainfall data, Gumble's distribution is done 2 years, 5 years, 10 years, 20 years, 30 years, 50 years and 100 year return period and shown in table 2.

Table 2: Rainfall Intensity chart of Khulna for different return period

Hours	**Intensity (2Year return period)**	**Intensity (5Year return period)**	**Intensity (10Year return period)**	**Intensity (20Year return period)**	**Intensity (30Year return period)**	**Intensity (50Year return period)**	**Intensity (100Year return period)**
24	5.12	7.73	9.45	11.10	12.06	13.24	14.85
48	3.45	5.00	6.03	7.02	7.58	8.29	9.25
72	2.63	3.84	4.64	5.41	5.85	6.40	7.15
96	2.19	3.13	3.75	4.35	4.69	5.12	5.70
120	1.89	2.65	3.16	3.64	3.92	4.27	4.74
144	1.67	2.33	2.76	3.18	3.42	3.72	4.13
168	1.56	2.15	2.54	2.92	3.13	3.40	3.77

From this table long duration IDF curve is generated for Khulna which is shown in Figure 2. From this figure, it is seen that the highest rainfall intensity is 14.85mm/hr for 100 year return period for duration of 24 hours. For the same duration but 2 year return period the rainfall intensity is noted 5.12 mm/hr.

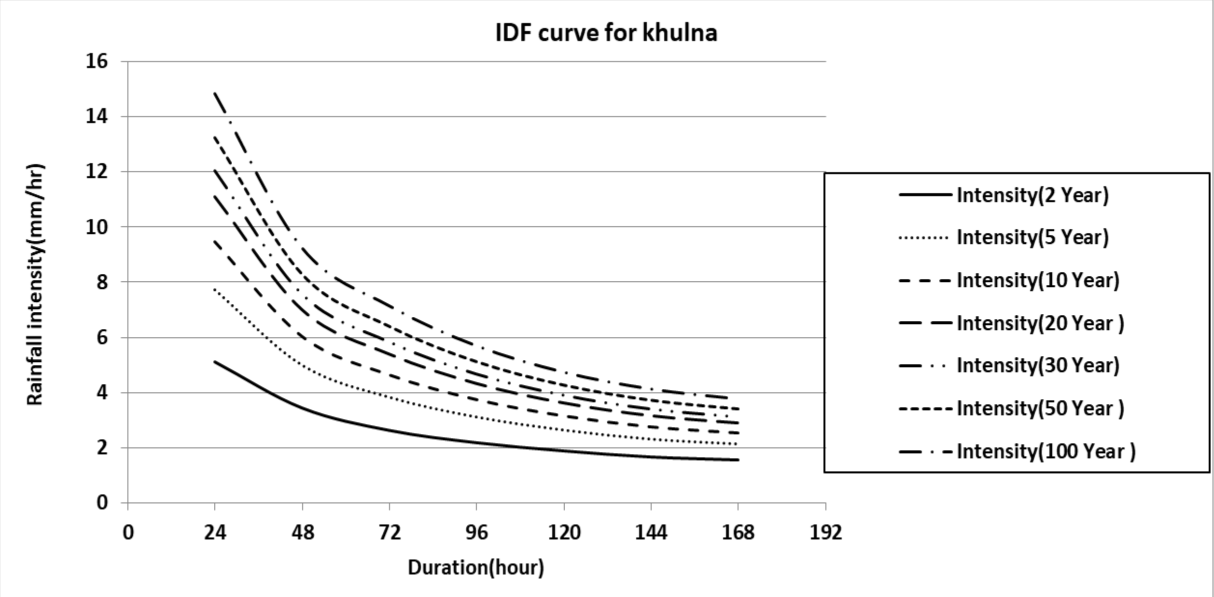

Figure 2 : IDF curve for Khulna for long duration data

3.3 Conversion of long duration data to short duration data using three empirical equations

The IDF formulas are empirical equations representing a relationship among maximum rainfall intensity (as dependent variable) and other parameters such as rainfall duration and frequency (as dependent variable). There are some commonly used formulas found in literature of hydrology applications (Chow et al., 1988). Some of the equations are shown in equation (9), (10), (11) which describe rainfall intensity duration relationship:

Talbot equation:

$$i = \frac{a}{d + b} \tag{9}$$

Sherman equation :

$$i = \frac{a}{(b + d)^e} \tag{10}$$

Kimjima equation:

$$i = \frac{a}{d^e + b} \tag{11}$$

Where i is the rainfall intensity(mm/hour), d is the duration(hour) and a, b, e are the constant parameters of related metrological conditions.

For Talbot, Sherman and Kimjima equation, the value of the constant parameters a, b,e are estimated using the long duration data for 2 year, 5 year, 10 year, 20 year, 30 year, 50 year, 100 year return period. From the estimated value of parameters for different return period, the rainfall intensity is converted to short duration data for 2 year, 5 year, 10 year, 20 year, 30 year, 50 year, 100 year return period. The results from Talbot and Sherman equation is shown in table 3 & 4.

Table 3 : Conversion of Short Duration data by Talbot formula for different return period

Hr	Intensity 2 RT	Intensity 5 RT	Intensity 10 RT	Intensity 20 RT	Intensity 30 RT	Intensity 50 RT	Intensity 100 RT
0.25	8.2461	13.5477	17.2110	20.7895	22.8693	25.4861	29.040
0.5	8.2451	13.5097	17.1587	20.7228	22.7939	25.3997	28.938
1	8.2432	13.4344	17.0551	20.5906	22.6448	25.2289	28.737
1.5	8.2413	13.3598	16.9528	20.4601	22.4975	25.0603	28.540
2	8.2394	13.1412	16.6531	20.0783	22.0670	24.5677	27.962
3	8.2356	12.7246	16.0843	19.3559	21.2537	23.6386	26.873
5	8.2280	12.3337	15.5532	18.6836	20.4981	22.7771	25.866
6	8.2243	11.9661	15.0559	18.0565	19.7945	21.9763	24.932
12	8.2016	9.6621	11.9896	14.2344	15.5296	17.1517	19.344
24	8.1568	7.4969	9.1839	10.8040	11.7366	12.9027	14.476

Table 4 : Conversion of Short Duration data by Sherman formula for different return period

Hr	Intensity 2 RT	Intensity 5 RT	Intensity 10 RT	Intensity 20 RT	Intensity 30 RT	Intensity 50 RT	Intensity 100 RT
0.25	89.5971	165.1596	218.1502	270.2414	300.6037	338.8674	390.907
0.5	58.1434	103.9776	135.7576	166.8429	184.9126	207.6463	238.511
1	37.7318	65.4599	84.4836	103.0062	113.7467	127.2385	145.527
1.5	29.2992	49.9366	64.0140	77.6867	85.6042	95.5418	109.001
2	24.4858	41.2108	52.5752	63.5944	69.9698	77.9674	88.793
3	19.0135	31.4380	39.8367	47.9625	52.6584	58.5448	66.507
5	13.8250	22.3538	28.0850	33.6162	36.8084	40.8067	46.210
6	12.3387	19.7921	24.7909	29.6113	32.3921	35.8742	40.579
12	8.0071	12.4602	15.4277	18.2816	19.9256	21.9825	24.759
24	5.1961	7.8445	9.6008	11.2868	12.2570	13.4701	15.107

3.4 RMSE value calculation

The RMSE value is estimated for Talbot, Sherman and Kimjima formula for all the mentioned return periods and the average RMSE values of the equations are shown in table 5.

Table 5: RMSE value comparison for different equations

Equation	RMSE value
Talbot	0.162724769
Sherman	0.093710522
Kimjima	0.176516145

From table 5, it is clear that the RMSE value is least for Sherman equation. From the RMSE value, it can be concluded that Sherman equation is best fitted for this region. The IDF curve using Sherman formula is shown for Khulna city in Figure 3. From this figure, it is seen that the highest rainfall intensity is 390.9 mm/hr for 100 year return period for duration of 0.25 hour. For the same duration but 2 year return period the rainfall intensity is noted 89.59 mm/hr.

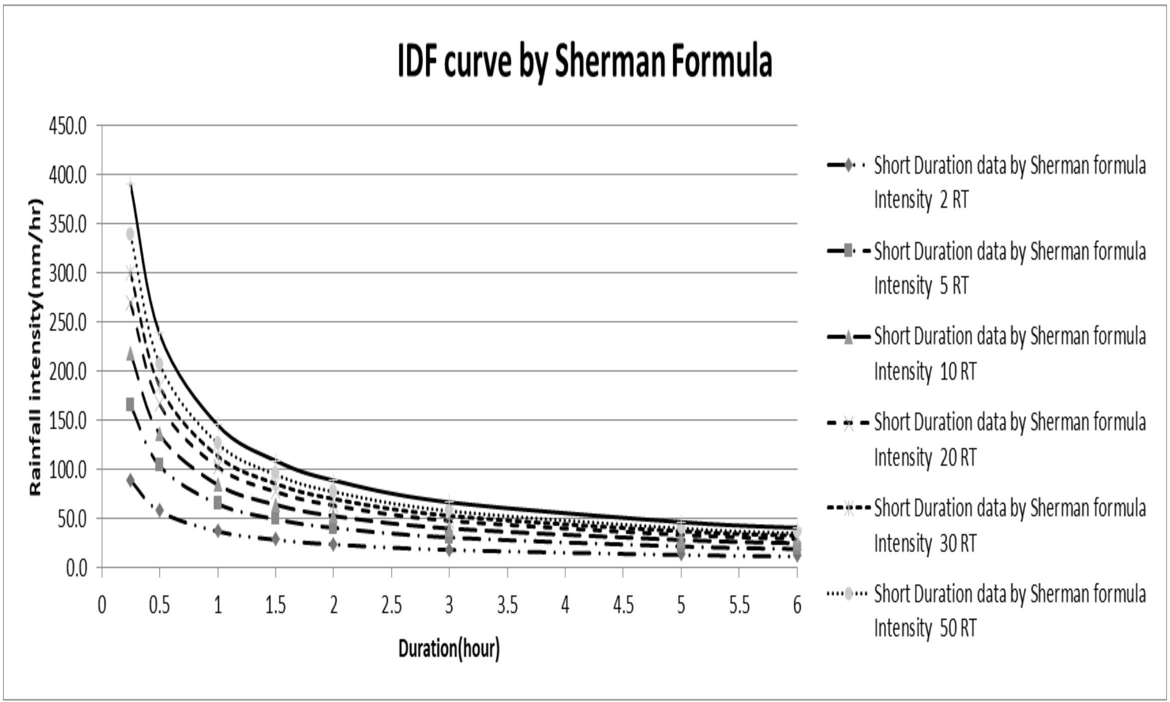

Figure 3 : IDF curve of Khulna using Sherman equation

Finally, a hyetograph of 2hr-5yr is generated for Khulna using the Sherman equation which is best fitted for this region. The hyetograph values are shown in table 6.

Table 6 : Generation of 2hr-5yr hyetograph for Khulna

Duration (min)	Intensity (mm/hr)	Cumulative Depth (mm)	Incremental Depth (mm)	Time	Precipitation (mm)
10	217.45	36.24	36.24	0-10	2.47
20	136.66	45.55	9.31	10-20	2.87
30	104.15	52.07	6.53	20-30	3.42
40	85.89	57.26	5.18	30-40	4.38
50	73.97	61.64	4.38	40-50	6.53
60	65.46	65.46	3.82	50-60	36.24
70	59.04	68.88	3.42	60-70	9.31
80	53.98	71.97	3.09	70-80	5.18
90	49.89	74.84	2.87	80-90	3.82
100	46.49	77.48	2.64	90-100	3.09
110	43.61	79.95	2.47	100-110	2.64
120	41.14	82.28	2.33	110-120	2.33

The 2hr-5yr hyetograph for Khulna city using the best fitted equation (Sherman equation) is shown in the following figure 4. In this figure it is seen that, the precipitation depth starts from 2.47 mm up to a peak of 36.24 mm then decreases to 2.33 mm.

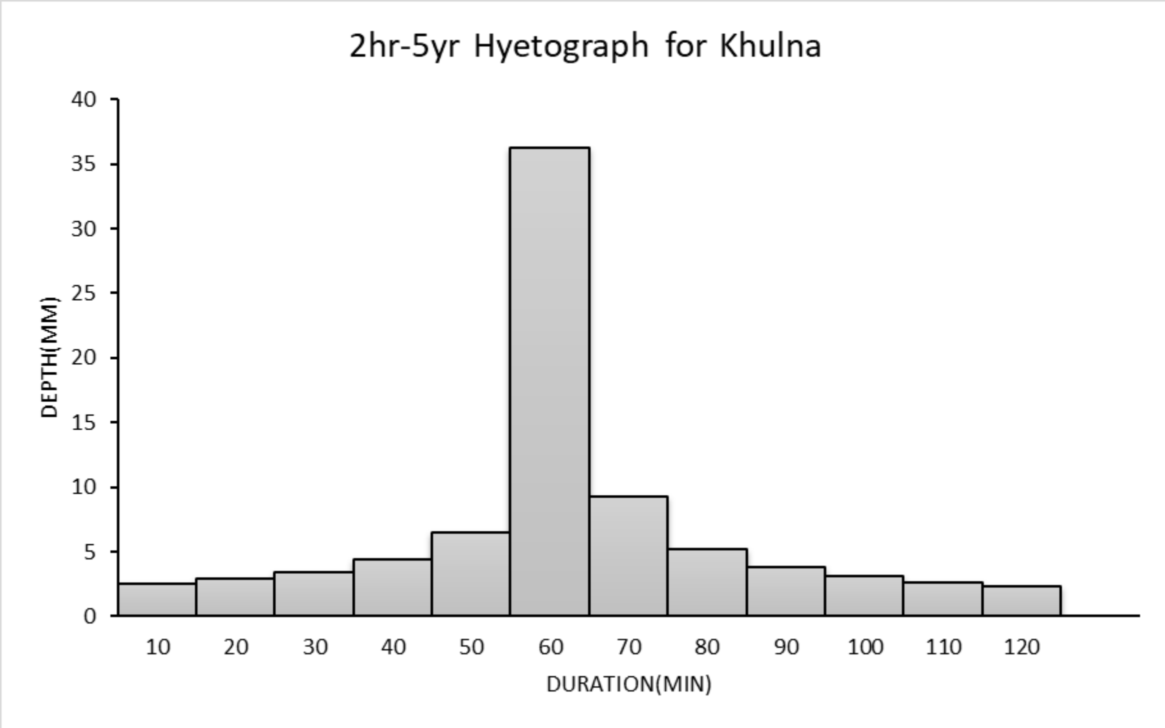

Figure 4 : 2hr-5yr hyetograph for Khulna

4. CONCLUSIONS

For drainage design and other practical applications, most hydrological studies require short duration rainfall data which are rare in developing countries. This study has been conducted to the formulation and construction of IDF curves of Khulna using daily rainfall data using the scaling properties with three different formulas. It is shown from the comparison of RMSE values that within these three formulas, Sherman formula is best suited for Khulna. Finally, a 2hr-5yr hyetograph is generated for the study area.

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