

## **NONLINEAR INELASTIC BEHAVIOR MODELING VIA BOUC-WEN MODEL FOR REAL-TIME CONTROL IMPLEMENTATIONS**

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### **ABSTRACT**

Often structures are experiencing large inelastic type deformation due to extreme external loads such as earthquake, wind, blast, and fire. However, the magnitudes and intensity of any dynamic input loads play a pivotal role in destruction of structures. Therefore, it is crucial to deal with those types of extreme dynamic loads. Further complicacy is induced by the vibration mitigation and controlling devices, especially, the smart devices those are accompanied with highly nonlinear behavior such as electromagnetic/magnetorheological damper. Hence, appropriate modelling for the aforementioned type inelastic and hysteretic behavior is needed. In order to achieve this goal, herein a prototype single-degree-of-freedom (SDOF) nonlinear system is considered which is coupled with Bouc-Wen type element. A numerical investigation is performed by using MATLAB and SIMULINK. The motivation behind the selection of Bouc-Wen model is due to its versatility and applicability for nonlinear behavior modeling and the performance of the early mentioned model. The preliminary results show that the studied is capable of predicting nonlinear inelastic behavior quite accurately. Furthermore, it is worth mentioning here that the conceptual framework is expected to be implemented for real-time control implementations. In order to implement the framework in real-time, the SIMULINK is employed as computing tool to solve the nonlinear system and the restoring force is estimated thereafter. In contrary, the offline simulations are conducted via the use of MATLAB. Moreover, the online results (e.g. form SIMULINK) are compared with offline results, achieving a good match between the online and offline results. Future study is planned to extend the framework for multi-storey system with limited sensor information for real-time vibration control implementations.

**Keywords:** *Nonlinear inelastic response, Bouc-Wen model, Real-time control, Hysteresis, Nonlinear system.*

## 1. INTRODUCTION

The structures and infrastructures (e.g. building, bridges) are playing a pivotal role in the national economy all over the world. The aforementioned structures are not only providing shelter or point-to-point transportation network connectivity but also making daily-life effortless. Therefore, any interruption or severe damage to those structures may lead to usability problem including financial loss as well as human lives. The damage is typically induced by extreme dynamic loads such as earthquake. In order to minimize the damage of early mentioned structures, modern structures are equipped with energy dissipative devices such as passive, active, semi-active or hybrid control systems (Miah, 2015). However, almost all of the control devices demand an appropriate controlling strategy or algorithm (except passive systems) that assist the device to perform optimally. In order to be effective, the aforementioned algorithms and devices need to work in real-time (Miah et al., 2017) as often the hitting time of any dynamic loading is unknown. One-step further, the complicacy is induced by the complex nonlinear behavior of the adopted devices (e.g. active, semi-active). Therefore, it is essential to understand and predict their behavior precisely in real-time for optimal controlling performances. An experimental investigation of semi-active by using H-infinity robust control algorithm was performed by (Cetin et al., 2011). The previously mentioned author used a magnetorheological damper (MR) as a controlling device to mitigate the vibration of a 6 degree-of-freedom (DOF) system and significant reduction of overall response was reported. In (Miah et al., 2017) presented real-time experimental verification of a novel control scheme that was proposed in their preceding work (Miah et al., 2015).

The smart vibration controlling devices are linked to complex nonlinear behavior which can be modelled by various model (Bhowmik, 2011). The Bouc-Wen model is one of the most commonly used nonlinear model that is used to predict hysteretic type nonlinear behavior (Bouc, 1967) and (Wen, 1976). The efficacy of the aforementioned model has been verified by many models such as (Baber and Noori, 1985; Baber and Noori, 1986; Bhowmik, 2011; Charalampakis and Koumoussis, 2008; Charalampakis and Koumoussis, 2009; Kottari et al., 2014; Song and Der Kiureghian, 2006). However, in a recent study carried by (Miah et al., 2015) shows that the conventional Bouc-Wen model itself may not be capable of capturing the complex behavior of a new disc-type MR damper.

As the modeling of the hysteretic nonlinear behavior is an unavoidable part of real-time control implementation system coupled with smart vibration mitigating devices. In a nutshell, this study focused on real-time implementation for vibration mitigation and control. The rest of the paper is organized as follows: the immediate section covered the mathematical modeling, the next section contains numerical simulations and results, and finally the outcome is summarized in conclusion.

## 2. MATHEMATICAL FORMULATION

### 2.1 The Bouc-Wen Model

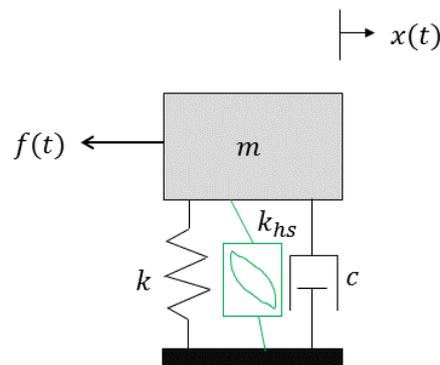


Figure 1: The structure with nonlinear element.

The numerical investigations are performed by considering a single-degree-of-freedom (SDOF) system, which is assumed to be attached to a nonlinear element. The sample structure linked with a nonlinear hysteretic type element is depicted in figure 1.

$$m\ddot{x} + c\dot{x} + \underbrace{\alpha k_i x + (1 - \alpha)k_i z_{hs}}_{F_{non}} = f(t) \quad (1)$$

The restoring (nonlinear) force is denoted by  $F_{non}$  in the equation (1),  $m$  is the mass of the system,  $x$  represents the displacement,  $c$  is the damping coefficient,  $z_{hs}$  represents the hysteretic displacement. In order to make it clearer, the nonlinear restoring force is given by,

$$F_{non} = \frac{k_f}{k_i} \left( \frac{F_y}{x_y} \right) x + \left( 1 - \frac{k_f}{k_i} \right) \left( \frac{F_y}{x_y} \right) z_{hs} \quad (2)$$

where  $z_{hs}$  represents the hysteretic displacement,  $\alpha$  is the ratio of the post-yield ( $k_f$ ) to pre-yield ( $k_i$ ), the pre-yield ( $k_i$ ) stiffness is given by

$$k_i = \left( \frac{F_y}{x_y} \right) \quad (3)$$

where  $F_y$  is the yield force and  $x_y$  is the yield displacement. In equation (2), the term hysteretic displacement ( $z_{hs}$ ) can be calculated via the equation given below,

$$z_{hs} = \theta \times \dot{x} - \beta \times |\dot{x}| \times |z_{hs}|^{(n-1)} \times z_{hs} - \gamma \times \dot{x} \times |z_{hs}|^n \quad (4)$$

where  $\theta$ ,  $\beta$ ,  $\gamma$  and  $n$  are the dimensionless quantities, typically, all of those quantities are responsible for the behavior of the model. Hence,  $\theta$ ,  $\beta$ ,  $\gamma$  and  $n$  requires appropriate tuning to obtain the optimal performance of the Bouc-Wen model.

## 2.2 The State-Space Formulation

A compact formulation is adopted for the offline implementations that are known as the state-space formulation (SSF). The main advantage of the SSF is that this formulation brings a second order differential equation (e.g. equation of motion) into a first order differential. As a result, it becomes much easier to solve and most importantly, one can get the response of the system only solving one equation called system/process equation. However, it needs to be mentioned that full-formulation of SSF contains two main equations: (i) system/process equation, shown in equation (5), (ii) observation/measurement equation, presented in equation (6). The process/system equation includes all of the information related to structure along with excitation/control force information. While the measurement equation deals with the desire measurement quantities such as displacement, velocity and acceleration. Designers have the freedom to formulate the measurement equation as per their problem specification and requirements.

Herein, the SDOF system has been re-formulated from the equation of motion by employing SSF as depicted below,

$$\dot{\chi} = A\chi + Bu \quad (5)$$

$$y = C\chi + Du \quad (6)$$

Assume the state variables are  $\chi_1 = x$ ,  $\dot{\chi}_1 = \dot{x} = \chi_2$ ,  $\dot{\chi}_2 = \ddot{x}$ , and  $\chi_3 = z_{hs}$ , hence the process equation can be written as,

$$\begin{Bmatrix} \dot{\chi}_1 \\ \dot{\chi}_2 \\ \dot{\chi}_3 \end{Bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -\alpha k_i m^{-1} & -c m^{-1} & (1-\alpha)k_i \\ 0 & \theta & 0 \end{bmatrix} \begin{Bmatrix} \chi_1 \\ \chi_2 \\ \chi_3 \end{Bmatrix} + \begin{bmatrix} 0 \\ m^{-1} \\ 0 \end{bmatrix} f + \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ -\beta & -\gamma \end{bmatrix} \begin{Bmatrix} |\chi_2| \times |\chi_3|^{(n-1)} \times \chi_3 \\ \chi_2 \times |\chi_3|^n \end{Bmatrix}}_{\text{Due to the Cross Term}} \quad (7)$$

And the accompanying measurement is defined as,

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \chi_1 \\ \chi_2 \\ \chi_3 \end{Bmatrix} \quad (8)$$

### 3. NUMERICAL EXAMPLES

#### 3.1 The Offline Investigations

The offline simulations are performed via the use of MATLAB. To achieve the goal, the state space formulation is adopted for the problem implementations. It needs to be mentioned that the offline solutions via MATLAB or any comparable computing tools are very common and quite useful, as they only required the fundamentals of programming skills.

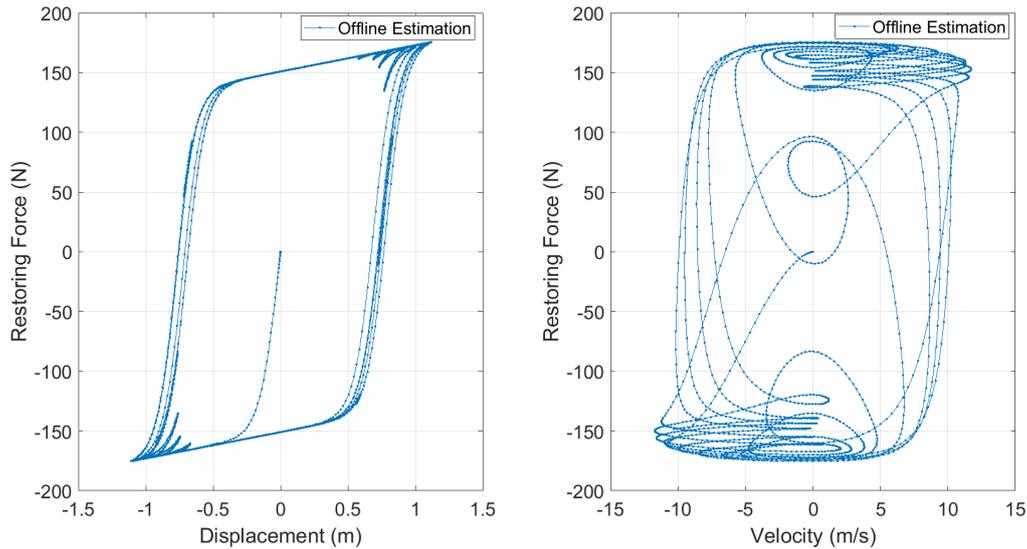


Figure 2: The offline results: restoring force versus displacement (left) and restoring force versus velocity (right).

In contrast, real-time implementations demand advanced computing tools (e.g. SIMULINK, dSPACE) as well as high computing power of the used computer such as parallel computing.

The outcome of the offline simulation is presented in figure 2. It can be observed from the early mentioned figure that the starting of the simulations/systems are moving to the steady-state conditions. Typically, displacement is smoother than the velocity as the later one is a time derivative of the displacement.

### 3.2 The Real-time Implementations

In order to investigate the real-time implementations herein, SIMULINK is employed as a real-time simulator tool along with MATLAB. An illustration of the SIMULINK model is furnished in figure 3. This implantation can be done via either MATLAB or SIMULINK independently or by using them both. In addition, the real-time implementations can be let it run for infinite time, while offline implementation is often done for a certain period due to cost-effectiveness analysis. In addition to those issues, real-time implementation also requires advanced knowledge of electro-mechanical. In other words, how the machine is going to behave or act with a real structure while they are built in different criterion.

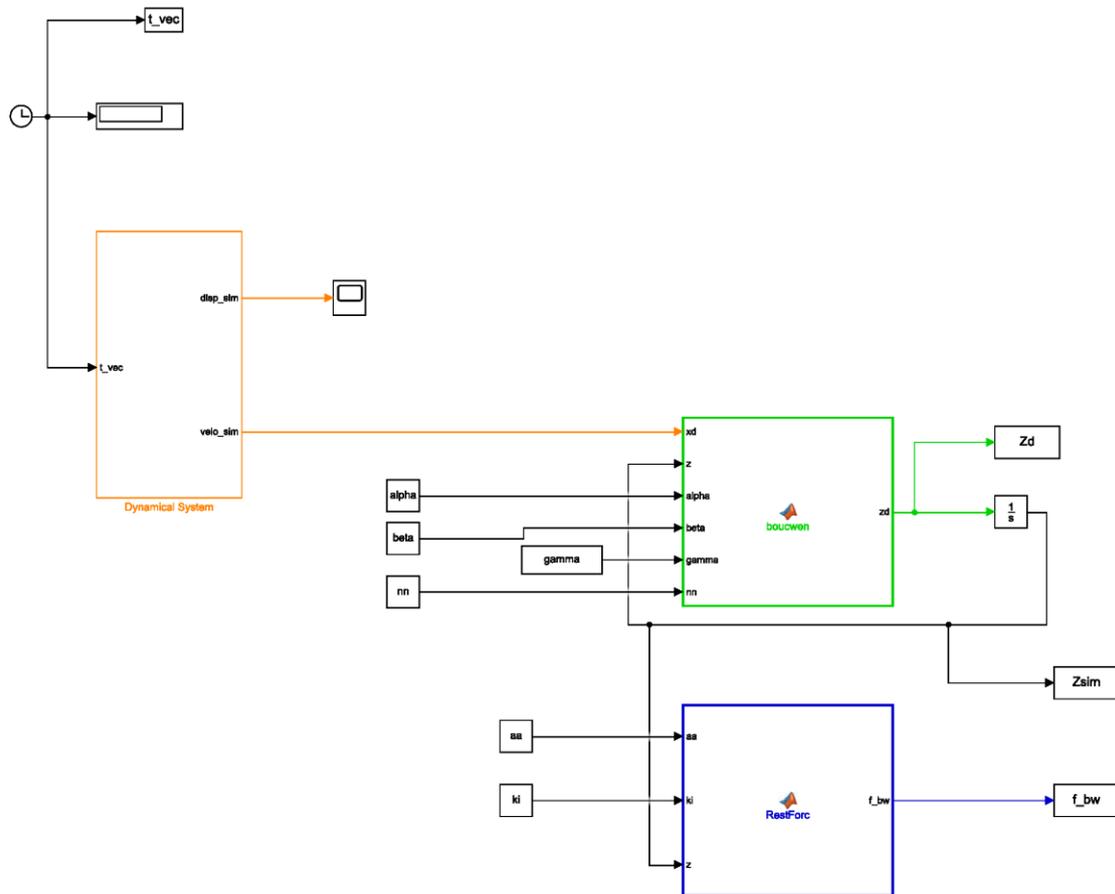


Figure 3: Illustrative model in SIMULINK.

In figure 4, it shows the sample results obtained from SIMULINK of the nonlinear model. This figure gives an idea how the system starts and goes into the steady-state level. However, it needs to be mentioned that the systems switching into steady state (see figure 4 (left)) situation fast, because the simulation is conducted for 5 seconds only (can be seen in figure 5).

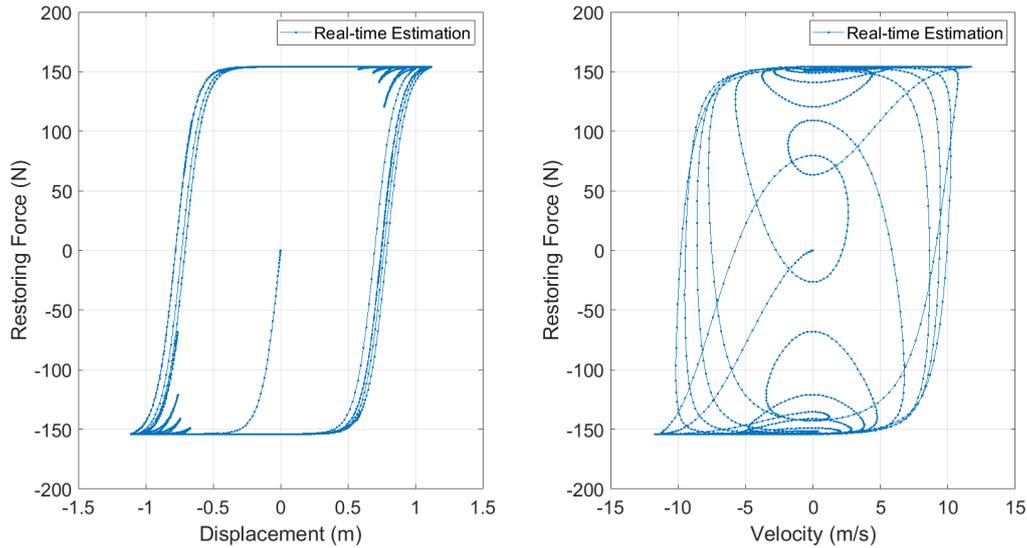


Figure 4: The real-time results: restoring force as a function of displacement (left) and velocity (right).

### 3.3 Results Comparison

For the comparison, the offline simulations are carried out by using MATLAB while the real-time simulations are performed via SIMULINK. In general, the offline simulation requires a longer time to complete the simulations in comparison to the real-time implementations e.g. SIMULINK. However, it also needs to be mentioned that this may not be true for all of the cases, as there will be many parameters those will control the overall simulation performance i.e., the complexity of the model itself, discretization, and ability of the used computer. The outcome of the offline and real-time results is compared in this section. Firstly, the displacement trajectory is compared in figure 5, it can be noticed that both the offline and real-time simulations results are almost identical.

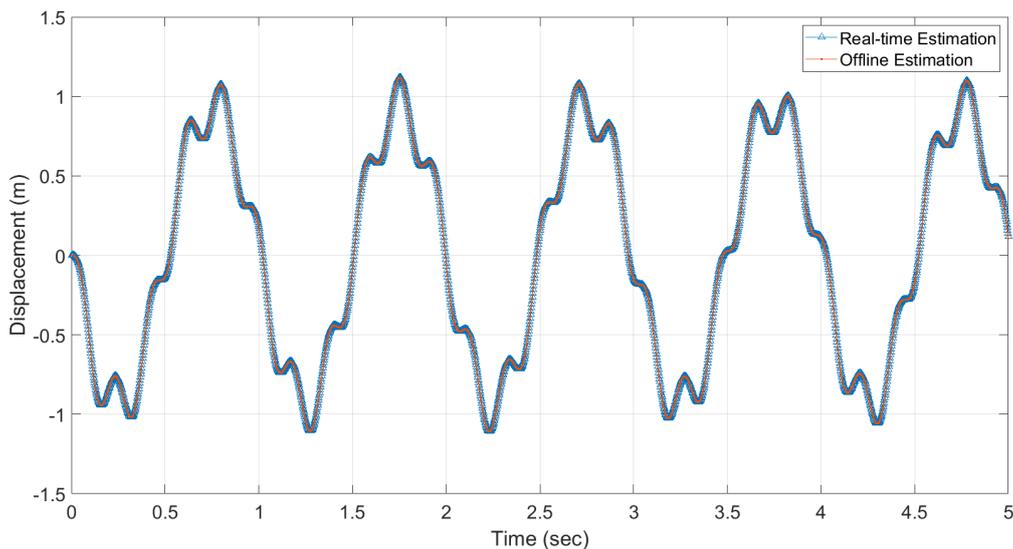


Figure 5: The full-time history of displacement.

However, even though, in the displacement trajectory, they agreed quite well that does not guarantee their hysteretic behavior with respect to the estimated restoring force. Moreover, this what can be seen in figure 6, where the restoring force hysteresis as a function of displacement is presented. In addition, it is even more visible in figure 7, where the restoring force hysteresis versus velocity is exhibited.

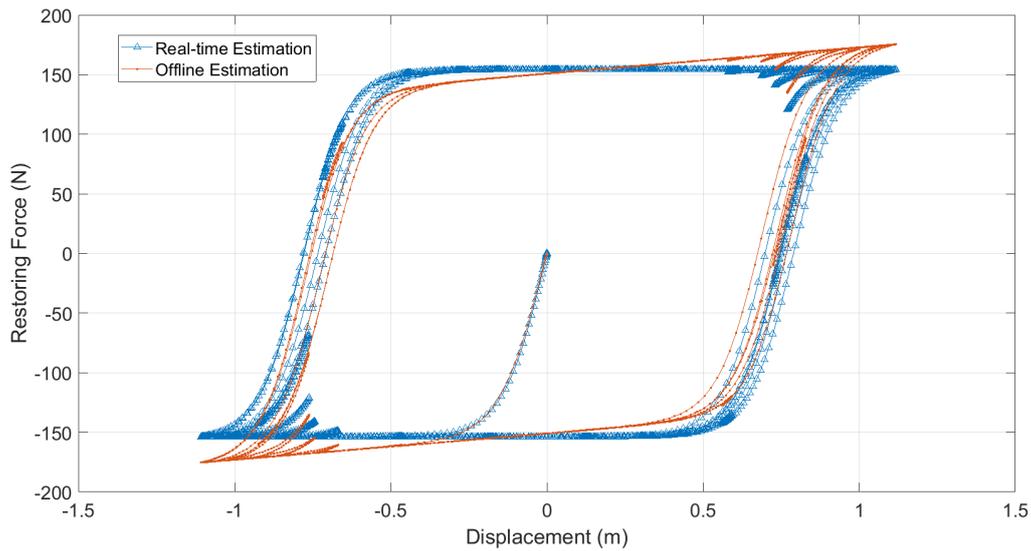


Figure 6: Restoring force hysteresis versus displacement..

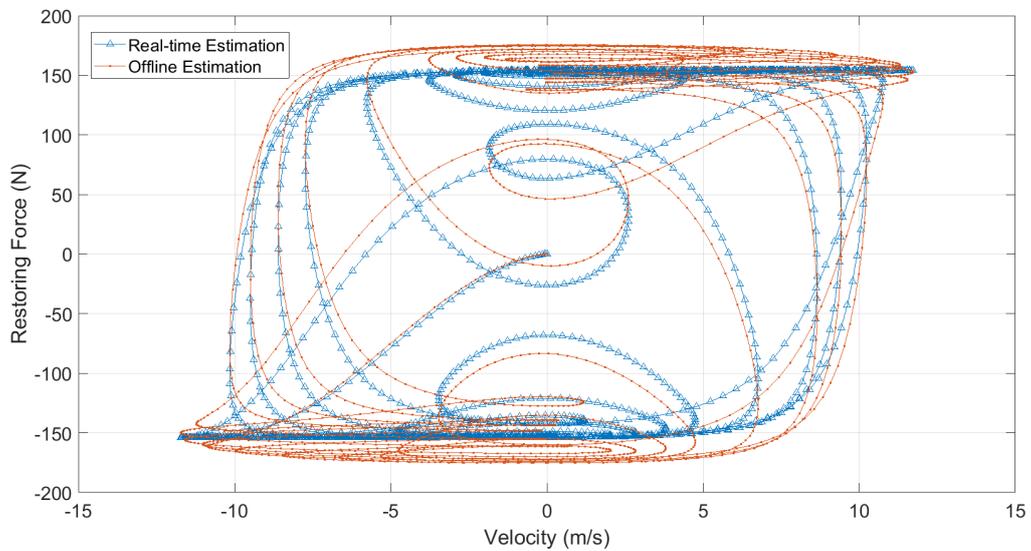


Figure 7: Evolution of restoring force hysteresis as a function of velocity.

Finally, the time-history of the restoring force is presented in figure 8. Moreover, from this figure, it can be clearly seen that indeed there is a little discrepancy between the offline and real-time estimation, especially, in the area of the peaks. In addition, this is what might be very important for the real-time implementations, as even such small error may create overall monitoring problem. Because the aforementioned error will then trigger the response of the systems.

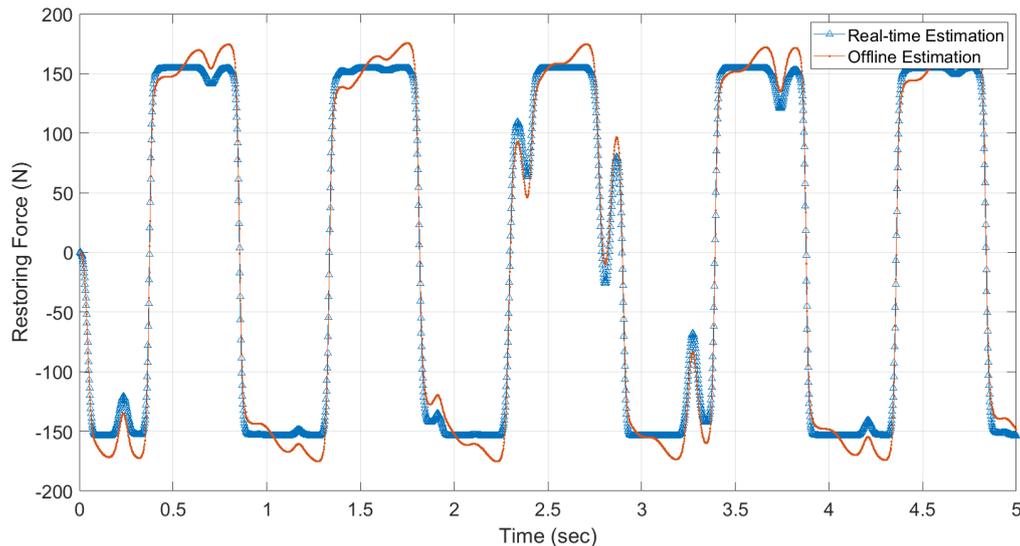


Figure 8: Time-history of the restoring force.

#### 4. CONCLUSIONS

Present study investigates the possibility of real-time modeling of nonlinear behavior via the use of Bouc-Wen model. The preliminary results show the potential of real-time modeling of smart type materials e.g. magnetorheological fluid which later can be implemented for the real-time vibration mitigation and control applications. The real-time performance is compared with the offline response. In addition, the outcome shows that both options can be suitable for modeling nonlinear behavior. However, as modern devices are needed to be operated in real-time, hence, the real-time model is inevitable. In order to be optimal in terms of overall performance, further study is essential to optimize the parameters of the adopted nonlinear model for more complex type behavior.

#### ACKNOWLEDGEMENTS

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