

## INTERPOLATION METHODS FOR GROUNDWATER LOWERING PREDICTION

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### ABSTRACT

Experiences around the world showed low water availability in many regions directly linked to reduced forest cover, soil degradation and on the other hand urbanization drastically affects local aquifer systems. Although, a major portion of the rapid urbanizing Chittagong city requirements are achieved through ground water extraction, however, due to lack of observation well and proper database there is no published record on possibilities of groundwater lowering in Chittagong city. Absence of continuous dataset on groundwater level, different interpolation methods are usually applied by researchers for prediction and those are inverse distance weighted (IDW), Spline and Kriging method of interpolation. In this study, the sum of least squares method under the time series regression analysis was used to compute groundwater level trend. Then Kriging method was used in GIS environment for interpolation of data and visual representation. This study conducted over 22 wells during 2009 to 2013, thus, the calculated water level lowering rate in the studied area was found as 0.12- 7.92 m reasonably matched with measure values. With propoer dataset, this method envisaged to provide necessary information to the decision support system.

**Keywords:** Lowering; Chittagong City; Kriging method

### 1. INTRODUCTION

Ground water is a precise and the most widely distributed resource of the earth. Ground water plays a vital role for drinking, irrigation, industrial purpose. Rapid increase of population is pushing usage of ground water into a limit. At present nearly one fifth of all the water used in the world is obtained from groundwater resources (Raghunath, 1987; Lee, Kim & Oh, 2012). Exploitation and over consumption of groundwater has increased in past years which causes decreased in groundwater levels and deterioration of water quality (Magesh, Chandrasekar & Soundranayagam, 2012). This study proposes to quantitatively evaluate and map the regional groundwater depletion in recent years. Chittagong is the second largest city in Bangladesh having the main sea port of the country. The total area of Chittagong and sub-urban areas is around 550 km<sup>2</sup>. The industrialized city of Chittagong is growing rapidly and the population in the City Corporation area has been estimated to be about 4.2 million. At present the demand for water supply within the city area is about 500 million liters per day (MLD). But, Chittagong Water Supply & Sewerage Authority (CWASA) has present capacity of supply about 210 MLD through its transmission and distribution pipelines. Out of The 210 MLD production capacity, 120 MLD productions are abstracted from groundwater and by local supply from some deep wells. Despite the abundance of water resources, there is a serious shortage of treated water supply to the city. The present shortfall of water supply is about 60% (CWASA, 2014).

Involvements of geostatistics is a useful tool for analyzing groundwater level depletion. Statistical methods for trend analysis vary from simple linear regression to more advanced parametric and non parametric methods (Helsel, Hirsch, 2002). To estimate the trend of groundwater level and the water level fluctuation, time series regression analysis was adopted using Microsoft office Excel 2007 (Shalini, Pandey & Nathawat, 2012). Previously

an attempt was made for Dhaka city using 17 years (1988-2004) information aimed to assess the water level fluctuation and predicting its trend using the MAKESENS computer model (Sarkar and Ali, 2009). Linear regression method was used to analyse the water level fluctuation in Kutahya – Cavdarhisar plain in Turkey (Nuri, 1988). Similar method and study was done in Kushtia district in Bangladesh (Adhikary, Chaki, Rahman & Gupta, 2012). Various characteristics controlling groundwater lowering can be presented and analyzed by using geographic information system (GIS). GIS technology is effective spatial tools widely used for the prediction, monitoring, management, and visual representation of geographic information (Lee, Kim & Oh, 2012; Oh, Kim, Choi, Park & Lee, 2011; Magesh, Chandrasekar & Soundranayagam, 2012; Shalini, Pandey & Nathawat, 2012). The Geographic Information System (GIS) presents an important tool in the effective management of groundwater resources (Shalini, Pandey & Nathawat, 2012). Different interpolation methods were adopted for surface interpolation (Shalini, Pandey & Nathawat, 2012, Gong, Mattevada & O'Bryant, 2014). Kriging method of Interpolation was used in mapping global solar radiation over southern Spain global solar radiation over southern Spain (Alsamamra, Arias, Vázquez & Pescador, 2009), estimating groundwater arsenic concentrations in Texas (Gong, Mattevada & O'Bryant, 2014), mapping liquefaction potential over alluvial ground (Pokhrel, Kuwano & Tachibana, 2013). In this study, the sum of least squares method under the time series regression analysis was used to compute groundwater level trend. Then Kriging method was used in GIS for interpolation of data and visual representation.

## 2. METHODOLOGY

### 2.1 Study area

The study area lies between 22°18'0 " to 22°24'0 " N latitude and 91° 46'0 " to 91° 51'0 " E longitude (Figure 1). Spatial distribution of Chittagong Sadar area reveals that thick prolific medium and coarse sand aquifer exist around the study area whose sediment formation is composed of medium to coarse sand. This aquifer is not laterally extended around the study area.

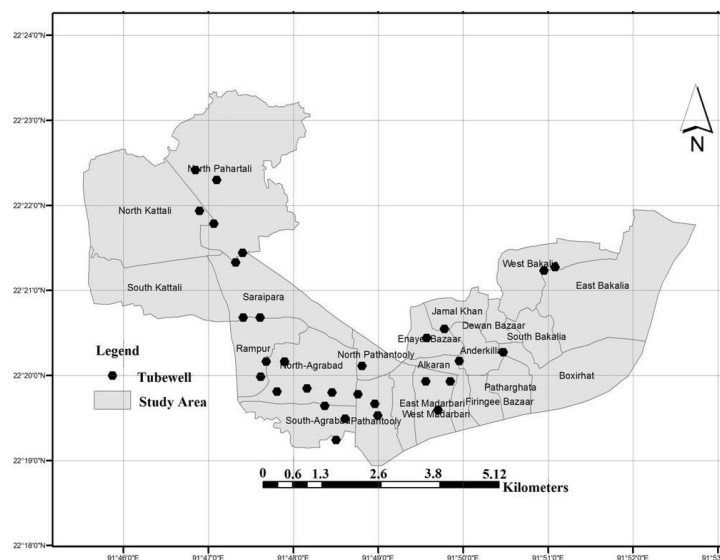


Figure 1: Study area and the selected well locations

Fine and medium sand aquifer also present around the study area. Aquifers in the Chittagong city are divided in to two separate zones, i.e., zone 1: Quaternary sediments form the aquifer in which the vertical recharge in deep aquifer is prohibited by the presence of

thick sequence of clay layer and rather horizontal recharge is dominant. But the zone 2 belongs to the higher elevated hills and valleys of exposed Tertiary sediments. Vertical recharge through percolation of rainwater is high in this area as the exposed areas are sandstone dominant. Apart from vertical recharge, horizontal recharge from the higher elevation to lower elevation may be responsible for recharging the groundwater in Zone 1. (Hossain, Bashar & Ahmed, 2008, CWASA, 2014).

Tertiary Aquifer System belongs to the northern part of the city area. Tertiary sediments are mainly exposed at surface and the ground level is very high with respect to the mean sea level. This part of the study area is the southern end of Sitakund anticline and stratigraphic layer of this part are dipping in east and west direction. Three distinct aquifers are mainly observed in all the area of this zone. But their thickness is highly variable latterly. Among the three aquifers, first or shallow aquifer is exposed at the surface and the average thickness of this aquifer is 23m and the maximum thickness is about 50m. Second aquifer situated at a depth of 140-150m and the average thickness is 27m. Third or deep aquifer is a good quality aquifer whose average thickness is about 35m. The strainer positions of all the production wells of this zone are situated in this aquifer and most of them are single screen. Fine to medium sand exist within the depth location ranges from 217 m to 266 m and finer materials that is clay exist from 266 m to 364 m (CWASA, 2014). Specific yield of these mentioned layers varies from 0.030 to 0.132, 0.030 to 0.132 and 0.030 in the study area. Up to 215m depth three aquifer layers have already been identified. These aquifer layers are semi-confined to confined in nature. The average range depth location and transmissivity value of these aquifer layers are presented below:

Table 1: Aquifer layer location and transmissivity value

<b>Aquifer Layer</b>	<b>Depth Location</b>	<b>Transmissivity Value</b>
<b>First Aquifer</b>	45m to 80m	148m <sup>2</sup> / day to 420m <sup>2</sup> / day
<b>Second Aquifer</b>	105m to 135m	308m <sup>2</sup> / day to 748m <sup>2</sup> / day
<b>Third Aquifer</b>	140m to 215m	697m <sup>2</sup> / day to 1040m <sup>2</sup> / day

## 2.2 Rainfall

Chittagong City is located in the tropical zone, which is subject to tropical climate. It is characterized by high temperature, and heavy rainfall with often-excessive humidity. There are three distinct seasons. The hot season continues from March to May but has some wet days. The monsoon season begins in June and continues usually to September with maximum temperature. The monsoon season generally comes and ends with cyclones. The cold and dry season begins in November and extends to February. Annual rainfall in the city ranges from 259.98 mm to 2540.8 mm in past nine years since 2004. Average annual rainfall is 2,557 mm (Table 2). Monthly average rainfall is very low during cold and dry season ranging from 9.7 mm to 57.4mm. Reversely, remaining seasons of hot and monsoon have comparatively abundant rainfall of from 87 mm to 594 mm per month on an average.

Table 2: Monthly Average Rainfall (mm) in Chittagong for the Period from January 2004 to January 2013 (Bangladesh Meteorological Department)

Month	2009	2010	2011	2012	2013
Jan					
Feb		7.87		17.27	0.5
March		154.18	109.72	13.46	3.3
April	75.95	31.49	49.28	226.31	60.19
May	374.39	373.63	299.46	228.34	759.96
June	431.81	648.46	456.43	754.38	572.52
July	1248.2	295.39	583.96	741.44	343.16
Aug	580.64	460.01	807.47	283.47	258.32
Sep	281.18	116.82	764.79	206.24	183.64
Oct	299.72	320.81	23.87		359.16
Nov	17.27	37.59		3.55	
Dec		19.05			
<b>Total</b>	<b>3309.1</b>	<b>2465.3</b>	<b>3095</b>	<b>2474.5</b>	<b>2540.8</b>

### 2.3 Well Flow Rates

Available bore log and flow details of 30 tube wells were collected from CWASA. Discharge rate is decreasing year by year and hence water level is increasing (Figure 2a and 2b).

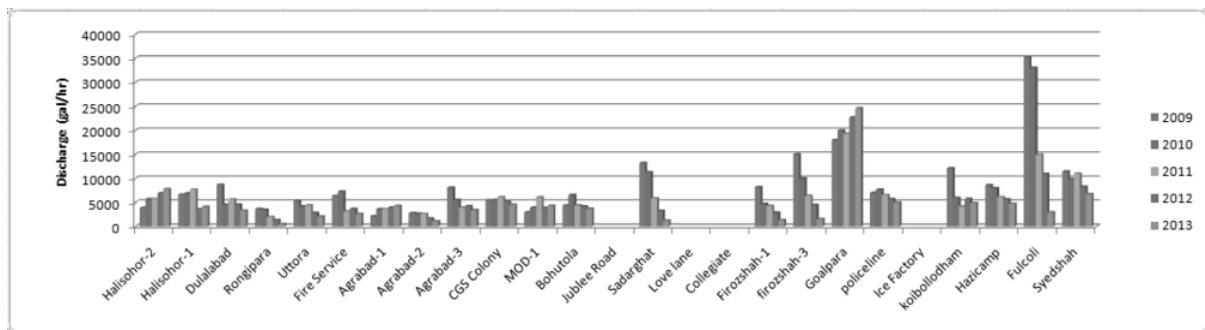


Figure 2 a: Locationwise discharge variation

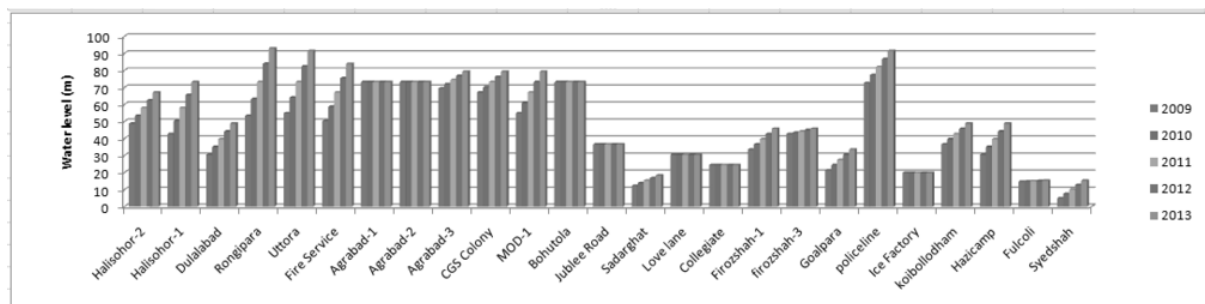


Figure 2 b: Locationwise water level variation

Geographic Information System (GIS). GIS technology is effective spatial tools widely used for the prediction, monitoring, management, and visual representation of geographic information. Different interpolation methods were adopted for surface interpolation (Shalini, Pandey & Nathawat, 2012; Gong, Mattevada & O'Bryant, 2014). Among them three methods of interpolation are widely use. The Inverse Distance Weighted (IDW) considers the concept

of spatial autocorrelation literally. This assumes that the unknown value of a point is influenced more by nearby control points than those further away. The nearer sample point provides information to the cell whose value is to be estimated, the more closely the cell's value will resemble the sample point's value. IDW performs reasonable with phenomena whose distribution depends on more complex sets of variables as this account only for the effects of distance. There is possibility to improve the accuracy of an IDW surface by using line layers as barriers. On elevation surfaces, barriers can represent abrupt changes in elevation, such as cliffs. Thin plate Splines creates surface that passes through control points and has the least possible change in slope at all points (Franke, 1982). In other words, thin-plate splines fit the control points with a minimum-curvature surface. Spline interpolation method fits a flexible surface, as if it were stretching a rubber sheet across all the known point values. The Spline method of interpolation estimates unknown values by bending a surface through known values. An advantage of the Spline interpolator is that it can make estimates outside the range of input sample points (Shalini, Pandey & Nathawat, 2012). Kriging method of Interpolation was used in mapping global solar radiation over southern Spain global solar radiation over southern Spain (Alsamamra, Arias, Vázquez & Pescador, 2009), estimating groundwater arsenic concentrations in Texas (Gong, Mattevada & O'Bryant, 2014), mapping liquefaction potential over alluvial ground (Pokhrel, Kuwano & Tachibana, 2013). In this study, the sum of least squares method under the time series regression analysis was used to compute groundwater level trend. Then Kriging method was used in GIS for interpolation of data and visual representation (Figure 3).

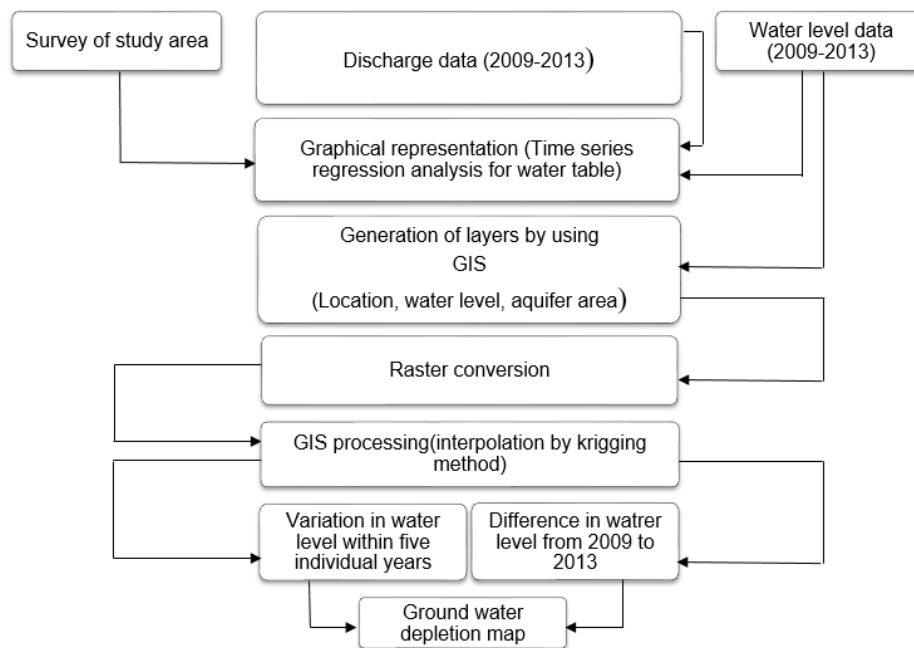


Figure 3: Adopted methodology

## 2.4 KRIGING

The basic idea of Kriging is to predict the value of a function at a given point by computing a weighted average of the known values of a function in the neighbourhood of the point. The method is mathematically closely related to regression analysis. Kriging refers to a family of least-square linear regression algorithms that attempts to predict values of a variable at locations of inadequate data. The description of kriging theory and its application are given in detail by Delhomme (1978). Ordinary kriging is the only technique that considers two sources of information regarding the attribute, the variation and the distance between points (Webster & Oliver, 2001; Alsamamra, Arias, Vázquez & Pescador, 2009). This is a single

method of incorporating characteristic irregular, small-scale variations into the construction of a contour map is to model the concentration field as a Spatial Random Field (SRF). The mathematics of SRFs is formidable. However, under certain simplifying assumptions, they produce classical linear estimators with very simple properties, allowing easy implementation for prediction purposes. These estimators, primarily ordinary kriging (OK), give both a prediction and a standard error of prediction at unsampled locations. This allows the construction of a map of both predicted values and level of uncertainty about the predicted values. Denote the SRF by  $Z(r)$ ,  $r \in D \subset \mathbb{R}^2$ . The following model for  $Z(r)$  is assumed:

$$Z(r) = \mu + \varepsilon(r) \quad (1)$$

Here,  $\mu$  is the fixed, unknown mean of the process, and  $\varepsilon(r)$  is a zero mean SRF representing the variation around the mean. In most practical applications, an additional assumption is required in order to estimate the covariance  $C_z$  of the  $Z(r)$  process. This assumption is second-order stationarity:

$$C_z(r_1, r_2) = E[\varepsilon(r_1) \varepsilon(r_2)] = C_z(r_1 - r_2) \quad (2)$$

This requirement can be relaxed slightly when you are using the semivariogram instead of the covariance. In this case, second-order stationarity is required of the differences  $\varepsilon(r_1) - \varepsilon(r_2)$  rather than  $\varepsilon(r)$  :

$$\gamma_z(r_1, r_2) = 1/2E[\varepsilon(r_1) - \varepsilon(r_2)]^2 = \gamma_z(r_1 - r_2) \quad (3)$$

By performing local kriging, the spatial processes represented by the previous equation for  $Z(r)$  are more general than they appear. In local kriging, at an unsampled location  $r_0$ , a separate model is fit using only data in a neighbourhood of  $r_0$ . This has the effect of fitting a separate mean  $\mu$  at each point. Given the  $N$  measurements  $Z(r_1), \dots, Z(r_N)$  at known locations  $r_1, \dots, r_N$ , you want to obtain an estimate  $\hat{Z}$  of  $Z$  at an unsampled location  $r_0$ . When the following three requirements are imposed on the estimator  $\hat{Z}$ , the OK estimator is obtained.

(i)  $\hat{Z}$  is linear in  $Z(r_1), \dots, Z(r_N)$ .

(ii)  $\hat{Z}$  is unbiased.

(iii)  $\hat{Z}$  minimizes the mean-square prediction error.  $E(Z(r_0) - \hat{Z}(r_0))^2$   
Linearity requires the following form for  $\hat{Z}(r_0)$ :

$$\hat{Z}(r_0) = \sum_{i=1}^N \lambda_i Z(r_i)$$

Applying the unbiasedness condition to the preceding equation yields

$$E\hat{Z}(r_0) = \mu \Rightarrow \mu = \sum_{i=1}^N \lambda_i E Z(r_i) \Rightarrow$$

$$\sum_{i=1}^N \lambda_i \mu = \mu \Rightarrow \sum_{i=1}^N \lambda_i = 1 \quad (4)$$

Finally, the third condition requires a constrained linear optimization involving  $\lambda_1, \dots, \lambda_N$  and a Lagrange parameter  $2m$ . This constrained linear optimization can be expressed in terms of the function  $L(\lambda_1, \dots, \lambda_N, m)$  given by

$$L = E(Z(r_0) - \sum_{i=1}^N \lambda_i Z(r_i))^2 - 2m \sum_{i=1}^N \lambda_i - 1 \quad (5)$$

Define the  $N \times 1$  column vector by

$$\lambda = (\lambda_1, \dots, \lambda_N)^T$$

and the (N+1) × 1 column vector  $\lambda_0$  by

$$\lambda_0 = (\lambda_1, \dots, \lambda_N, m)^T = \begin{pmatrix} \lambda \\ m \end{pmatrix}$$

The optimization is performed by solving

$$\frac{\partial L}{\partial \lambda_0} = 0$$

in terms of  $\lambda_1, \dots, \lambda_N$  and  $m$ .

The resulting matrix equation can be expressed in terms of either the covariance  $Cz(r)$  or semivariogram  $\gamma_2(r)$ . In terms of the covariance, the preceding equation results in the following matrix equation:

$$C\lambda_0 = C_0$$

Using this solution for  $\lambda$  and  $m$ , the ordinary Kriging estimate at  $r_0$  is

$$\hat{Z}(r_0) = \lambda_1 Z(r_1) + \dots + \lambda_N Z(r_N)$$

with associated prediction error. Where  $c_0$  is  $C_0$  with the 1 in the last row removed, making it an  $N \times 1$  vector.

### 3. OUTCOMES

Kriging of groundwater levels was applied in this study. The measure of the degree of spatial dependence among the sampled known points is the semi variance that can be fitted with a mathematical function or a model such as spherical, circular, exponential, linear, and Gaussian. These three methods are applied in this study for interpolation and then the errors were calculated (Table 3). This is evident that water table is declining per year in selected areas (Figure 4 and Table 4). In this study adopted interpolation method worked based on the collected details from the CWASA during 2009 to 2013. This study conducted over 22 wells during 2009 to 2013, thus, the simulated water level lowering rate in the studied area was found as 0.12- 7.92 m reasonably matched with measure values.

Table 3: Comparison of interpolation errors

Interpolation methods	Values in meter	Actual value	Error (%)
IDW	50.3	50.29	0.02
Spline	50.14	50.29	-0.29
Kriging (Ordinary)	50.13	50.29	-0.3.

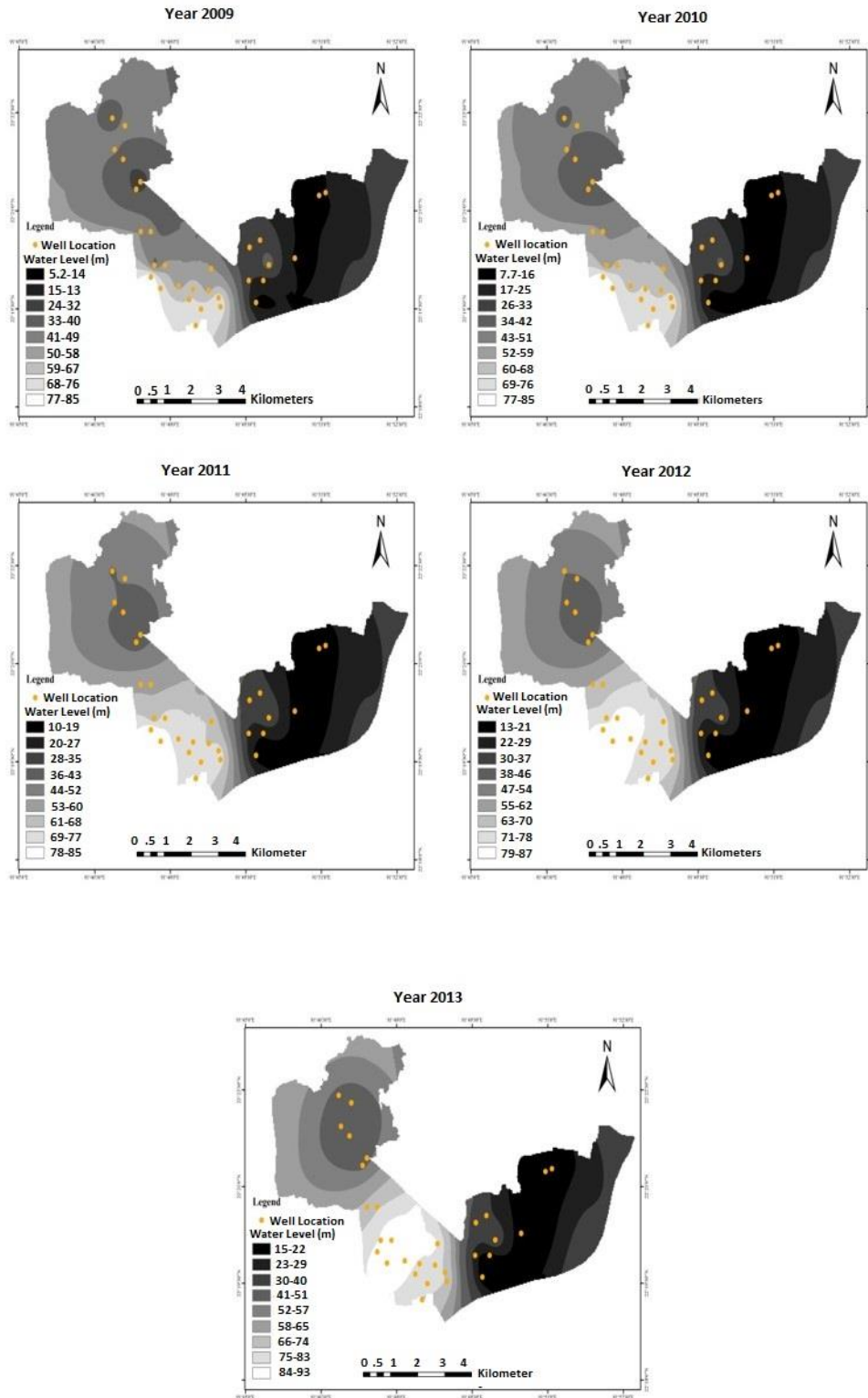


Figure 4: Map showing variation in water table in selected wells from 2009 to 2013.



Table 4: Water level declining rate in selected wells

Pump Name	Decline rate per year	Pump Name	Decline rate per year
Halisohor-2	3.656	Sadarghat	1.218
Halisohor-1	6.094	Love lane	0
Dulalabad	3.656	Collegiate	0
Rongipara	7.922	Firozshah-1	2.436
Uttora	7.312	firozshah-3	0.608
Fire Service	6.704	Goalpara	2.438
Agrabad-1	0	policeline	3.776
Agrabad-2	0	Ice Factory	0
Agrabad-3	1.948	koibollodham	2.436
CGS Colony	2.436	Hazicamp	3.656
MOD-1	4.874	Fulcoli	0.122
Bohutola	0	Syedshah	2.072
Jublee Road	0		

#### 4. CONCLUSIONS

This study comprises of an interpolation method to predict ground water lowering in the selected wells in Chittagong city. In this connection only few available wells were studied due to lack of observation information. The prediction showed on an average the lowering rate in the study area extends 0.12- 7.92 m to mimic the measured values. The prediction reasonably matches the observed data. With intensive field study, this is expected to provide reasonable information for the planners, policy makers, researchers to take necessary actions to ensure the optimum level of ground water table.

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