

## NUMERICAL MODELLING OF ANCHOR FOUNDATION IN DENSE SAND

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### ABSTRACT

Anchors and anchoring systems are widely used in different structures with different embedment, size, and shapes. The Finite Element Method has the ability to handle complex soil stratigraphy and it also has the potentiality of solving different soil-structure interaction problems. Presently evolved, almost all of the prevailing sophisticated models are very complex and incomplete in the sense that they do not define important factors such as strain localization, strain softening, etc. Above important factor overlooked in most of the current geotechnical models is a strong link between the model and a reliable set of experimental data. Mohr-Coulomb failure surface has corners or singularities, and therefore it is not mathematically convenient to use particularly for 3D problems because of discontinuities of gradient occur at edges and tip of the hexagonal shape yield surface pyramid. In this study, different approximation models are used to determine the appropriate models with respect to experiment. Furthermore, parametric studies are done on peak frictional and dilatancy angles. This study validated the rigorous 3D FE models, incorporating simple strain softening law for anchor foundation in dense sand using in-house finite element program. DP compromise cone predictions are found to be in better agreement with the experimental results. Dilatancy of sand makes significant effect on the uplift behavior of anchors. The greater dilation angle is resulted from the higher collapse load and displacement. With the increase of friction angle and embedment ratio, the effect of dilatancy is more remarkable.

**Keywords:** Anchors, approximation, dilatancy, finite element method, frictional angle

### 1. INTRODUCTION

Anchors are foundation systems that are designed primarily to resist uplift (tensile) loads. Anchor usage was started in early 1930's and with a boom after the Second World War and today, anchoring is a well-established branch of Geotechnical engineering. Different types of anchors and anchoring systems are being used widely all over the world and engineers are showing more enthusiasm on anchor usage techniques, their behaviour, and design. Anchor problems are related to different types of geotechnical and civil engineering constructions such as retaining walls, television and transmission towers, anchor bulkheads, submerged pipelines, offshore platforms, free-standing and guyed lattice towers, tension cables for suspension bridges, tent type roofs, and tunnels with different embedment, size, and shapes. Due to lack of proper installation technology and practical designers and/or engineers do not accept new technology; there is limited use of anchor foundations in Bangladesh. Anchor foundations have good potential in Bangladesh which can be used as tension members, earth reinforcement, and so on, which can reduce the cost of foundation of different structures significantly. Various studies of anchors have been conducted by numerous researches (Baker & Konder, 1966; Ball, 1961; Clemence & Veesaert, 1977; Davie & Sutherland, 1977; Deshmukh et al., 2010; Dickin, 1988; Hanna et al., 1972; Majer, 1955; Matsuo, 1967; Merifield & Sloan, 2006; Meyerhof & Adams, 1968; Mors, 1959; Murray & Geddes, 1987; Ovesen, 1981; Rokonuzzaman & Sakai, 2011; Rowe & Booker, 1979; Rowe & Davis, 1982; Sakai & Tanaka, 1998, 2007, 2009; Vermeer & Sutjiadi, 1985; Vesic, 1969). Some of the researchers are widely used limit state theories, which are based on a rigid elastic-perfectly plastic assumption which cannot adopt the failure in the real soil which

is highly progressive. So, the displacement based elastoplastic finite element method (FEM) is widely being used by the geotechnical researchers and engineers to find the load-displacement relationship and, thus, the collapsed load, due to its robust ability to deal with the complicated geometry and loadings, nonlinear constitutive law, anisotropic nature of soils, etc. The widely used model in elastoplastic geotechnical analysis is Mohr-Coulomb yield criterion. While this yield criterion is superseded by more complicated soil models for advanced applications, is widely used in the geotechnical analysis. The most important advantages of this model are its simplicity and the finite element solutions can be compared with different classical plasticity equations which is more useful for validating finite element codes (Abbo & Sloan, 1995). Rowe and Davis (1982) applied elasto-plastic FEM with Mohr-Coulomb failure criteria and continuous dilatancy, producing highly conservative design charts. Vermeer and Sutjiadi (1985) used non-associated elasto-perfectly plastic FEM model (Borst & Vermeer, 1984) to validate their proposed design equations. Merifield and Sloan (2006) used elasto-perfectly plastic FEM with Mohr-Coulomb yield criteria to compare their results obtained by limit state theories. Mohr-Coulomb failure surface has corners or singularities, and therefore it is not mathematically convenient to use particularly for 3D problems because of discontinuities of gradient occur at edges and tip of the hexagonal shape yield surface pyramid. It is well known that these singularities often cause stress integration schemes to perform inefficiently or fail. Originally the corners and the apex of the Mohr-Coulomb yield surface caused problems in the numerical implementation, so an approximate yield surface with smoothed or rounded corners has to be used. The singularities in the yield surfaces, where the gradient with respect to the stresses is undefined, occur at  $\theta = \pm 30^\circ$ . To deal with these singularities which are often encountered in finite element analysis, a satisfactory method is needed under the conditions of axial symmetry (Sloan & Booker, 1986). Nowadays, methods for implementing the corners explicitly exist, but the use of the rounded surfaces is still widespread. The implications of using these approximations, however, are not documented in the literature. Various techniques for dealing with these corners have been discussed by Hinton and Owen (1986); Sloan and Booker (1986); Zienkiewicz and Pande (1977). Drucker-Prager model has been proposed by Drucker and Prager (1952) as a smooth approximation to the Mohr-Coulomb law. The Drucker Prager model can also be made either more or less conservative than the Mohr-Coulomb law by fitting it to the inner or outer apices (Mijangos & O'Kelly, 2009). For non-associative flow rule, the major symmetry of the consistent tangent operator is lost. Further, cone apex is singular, and the normal to the potential surface is not defined. Special algorithmic treatment is needed around this region (Hofstetter & Taylor, 1991). In order to approximate the Mohr-Coulomb hexagon on the deviatoric stress plane ( $\pi$  plane), several strategies are available for determining Drucker-Prager cone parameters. According to author knowledge, no researcher has proposed a rigorous numerical model which can consider the combined effect of peak frictional angle, dilatancy, and singularities for the rectangular anchor foundations buried in the dense sand. So, in this study, an elasto-plastic 3D FEM model, incorporating non-associated simple strain softening constitutive law with shear-band, is validated against model tests to simulate uplift load-displacement relationships of rectangular anchor foundations and also determine the effect of peak frictional angle, dilatancy and Drucker-Prager approximation on the uplift behaviour of the shallow rectangular anchor in sand.

## 2. METHODOLOGY

In this study, an in-house finite element programs written in FORTRAN which is coded by Professor Tanaka is used. 3D FEM model is validated using simple strain softening law, for the vertically uploaded rectangular anchor foundations embed in dense sand for clearly understanding the related failure mechanisms. Furthermore, different approximate models to the exact Mohr-Coulomb model in the in-house finite element programs are implemented.

Besides, design graphs are developed for the practical design based upon detailed parametric studies on peak frictional and dilatancy angles.

## 2.1 Physical Modeling

The experimental setup, which is used for the experiments, is shown in Figure 1. The experimental results are used in this study with taking the permission from Sakai and his research group conducted at Mie University. The detailed procedure can be found from Rokonuzzaman and Sakai (2011).

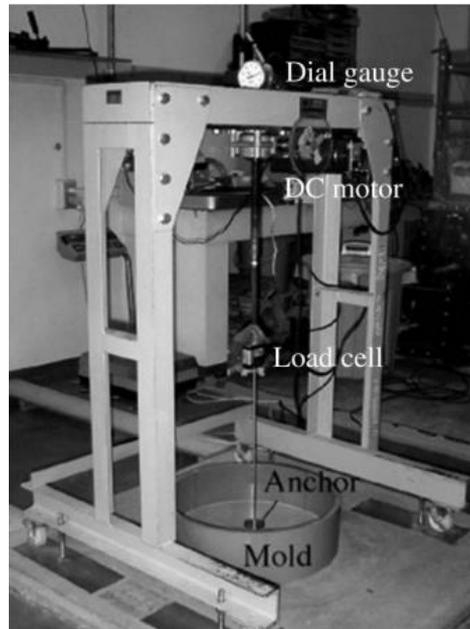


Figure 1: Setup for the anchor pullout experiment (Rokonuzzaman & Sakai, 2011)

### 2.1.1 Numerical Modeling

The 3D FE model uses the nonlinear elasticity (Hardin & Black, 1968), non-associated flow rules (Drucker-Prager potential and Mohr-Coulomb yield surface), simple strain softening constitutive law (Walters & Thomas, 1982) in an elasto-plastic framework. The dynamic relaxation method devised with return mapping algorithm (Sakai & Tanaka, 2009; Tanaka & Kawamoto, 1988) is used for the fast solution of highly nonlinear equations (e.g. dense Toyoura sands with the high frictional angle of 45°), which is very essential for 3D problems to save computational time. The standard FE solutions of strain-softening material are strongly meshed size-dependent. Several techniques have been proposed to resolve the mesh-dependent pathology of FE solutions; Pietruszczak and Mroz (1981) proposed the idea of employing a softening modulus scaled by the element size, which is used here.

A characteristic photograph of the ground surface is shown in Figure 2. It has two symmetrical vertical planes passing through the center of the foundation, validating the necessity of 3D design consideration. Also, taking the benefits of this symmetry, one-quarter of the domain is discretized into finite elements; some meshes are shown in Figure 3. The mesh extends following the recommendation of Bray et al. (1989). Zero-displacement boundary conditions are applied to prevent the out of plane displacements of the central vertical symmetrical boundaries and the base of the mesh was fixed in all three coordinate directions, except the anchor plate area. The differential quasi-elastic displacements are applied along the anchor boundary in small consecutive increments till to the failure, and the corresponding nodal forces were averaged over the displacement control nodes to find the uplift load. To construct the mesh, the same type of element, and the equivalent boundary conditions, soil conditions, and analysis procedure are maintained. Following the

recommendations of Lindsay (1980), the displacement ( $\delta$ ) and the uplift load ( $P$ ) are normalized as the displacement factor,  $\delta/B$ , and the pullout resistance factor,  $N_p (=P/\gamma_d HBL)$ , respectively. The material parameters are used for the analysis is shown in Table 1.

Table 1: Parameters for Simple Strain Softening Material Models

Parameters	Properties
Density ( $\text{g/cm}^3$ )	1.63
Void ratio, $e$	0.62
Relative density, $D_r$	0.95
Coefficient of shear modulus, $G_0$	500
Peak frictional angle, $\phi_p$	45
Dilatancy angle, $\psi$	20
Residual frictional angle, $\phi_r$ (degrees)	33
Poisson's ratio	0.3
B	0.5
C	0.5
D	0.5

Such a numerical model incorporating softening law must be verified before to apply for the analysis of anchor foundations, as the FE solutions can be sensitive to mesh size. For this purpose, cubic hexahedral elements finite elements of 14208 (Figure 3a) and 6525 (Figure 3b) with the size of 5 mm and 6.25 mm in the central zone, respectively, are used for an anchor foundation ( $H/B=2$ ,  $L/B=2$ ,  $B=50$  mm). The curves in the Figure 3c depict the relationship between the numerical pullout resistance and displacement factor, and it is shown that the numerical solutions are not susceptible to the mesh size effect.

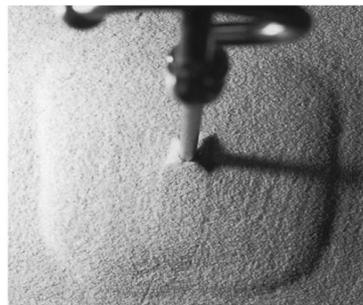


Figure 2: Characteristic photograph of heave on ground surface (square anchor)

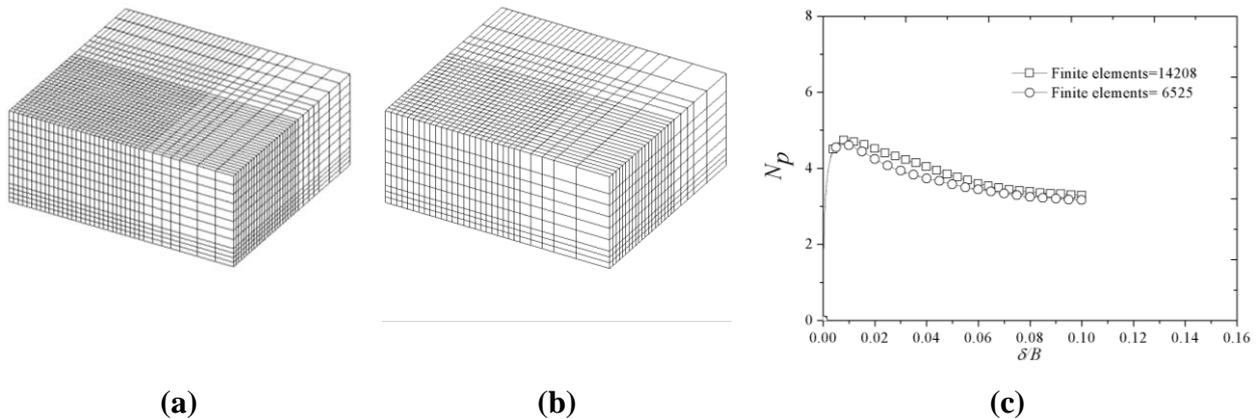


Figure 3: Mesh-size effect ( $H/B=2$ ,  $L/B=2$ ,  $B=50$  mm): (a) fine mesh (elements=14208), (b) course mesh (elements=6525), and (c) uplift resistance-displacement factor relationship

### 3. RESULTS AND DISCUSSIONS

#### 3.1 Validation and Verification with Experiment

The curves are shown in Figure 4, depict the experimental and numerical relationships between the pullout resistance and displacement factor of the square and rectangular anchor foundations with  $L/B= 1, 2, 3, 4, 5$  and  $6$  ( $B = 50$  mm and  $H/B= 2$ ). The experimental and numerical results show close agreement. All the curves show three distinct phases: the initial phase with the sharp increase in pullout resistance with the anchor displacement, followed by a softening nature of decreasing the pullout resistance with anchor displacement, and, finally, the pullout resistance remains unchanged with the further uplifting of the anchor, defining the residual phase. The overall shape of load-displacement curves is attributed to the progressive failure. The rate of softening after the peak uplift resistance factor is decreasing with the increase of  $L/B$ .

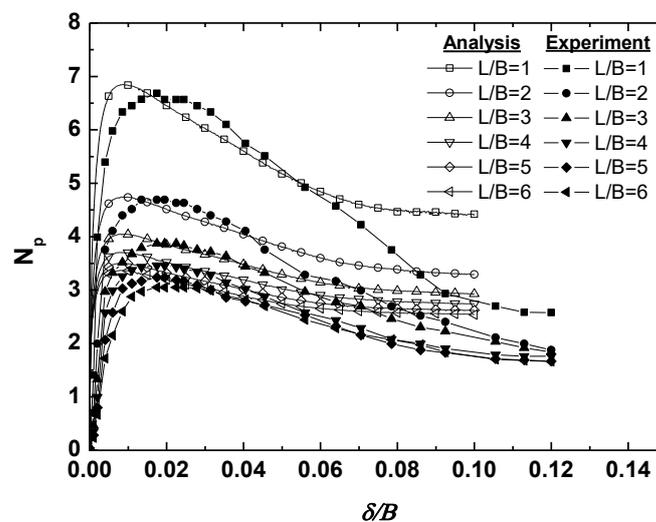


Figure 4: Uplift resistance-displacement factor relationships at  $H/B=2$

#### 3.2 Effect of Peak Frictional Angle and Dilatancy

Geotechnical material is non-associated material in apply, so non-associated flow rule ought to be adopted in soil model. Sand dilatancy depends on density and stress level. Dense sand with low-stress level exhibits shear dilation and loose sand with high-stress level

usually behaviors shear contraction. The dilation angle of zero degree corresponds to a soil that deforms plastically with zero volume change. This can be an affordable assumption for loose sands (Rowe & Davis, 1982). The uplift behavior of horizontal circular anchors with H/B varying from 1 to 3 have been simulated where  $\phi$  ranges from 25° to 45°,  $\psi$  from 0° to 25°. Figure 5 show the values of  $N_{pu}$  for H/B= 1, 2, 3 and various sand properties. The uplift capacity increases with dilation angle significantly and the effect of dilatancy become greater with the increase of friction angle and embedment ratio. When H/D=1,  $\phi=25^0$  and  $45^0$ , the uplift capacity for the case that  $\psi= 25^0$  are about 1.1 and 1.3 times of the values for the case that  $\psi= 0^0$ , respectively. When H/D=3,  $\phi= 25^0$  and  $45^0$ , the uplift capacity for the case that  $\psi= 30^0$  are about 1.2 and 1.5 times of the values for the case that  $\psi=0^0$ , respectively.

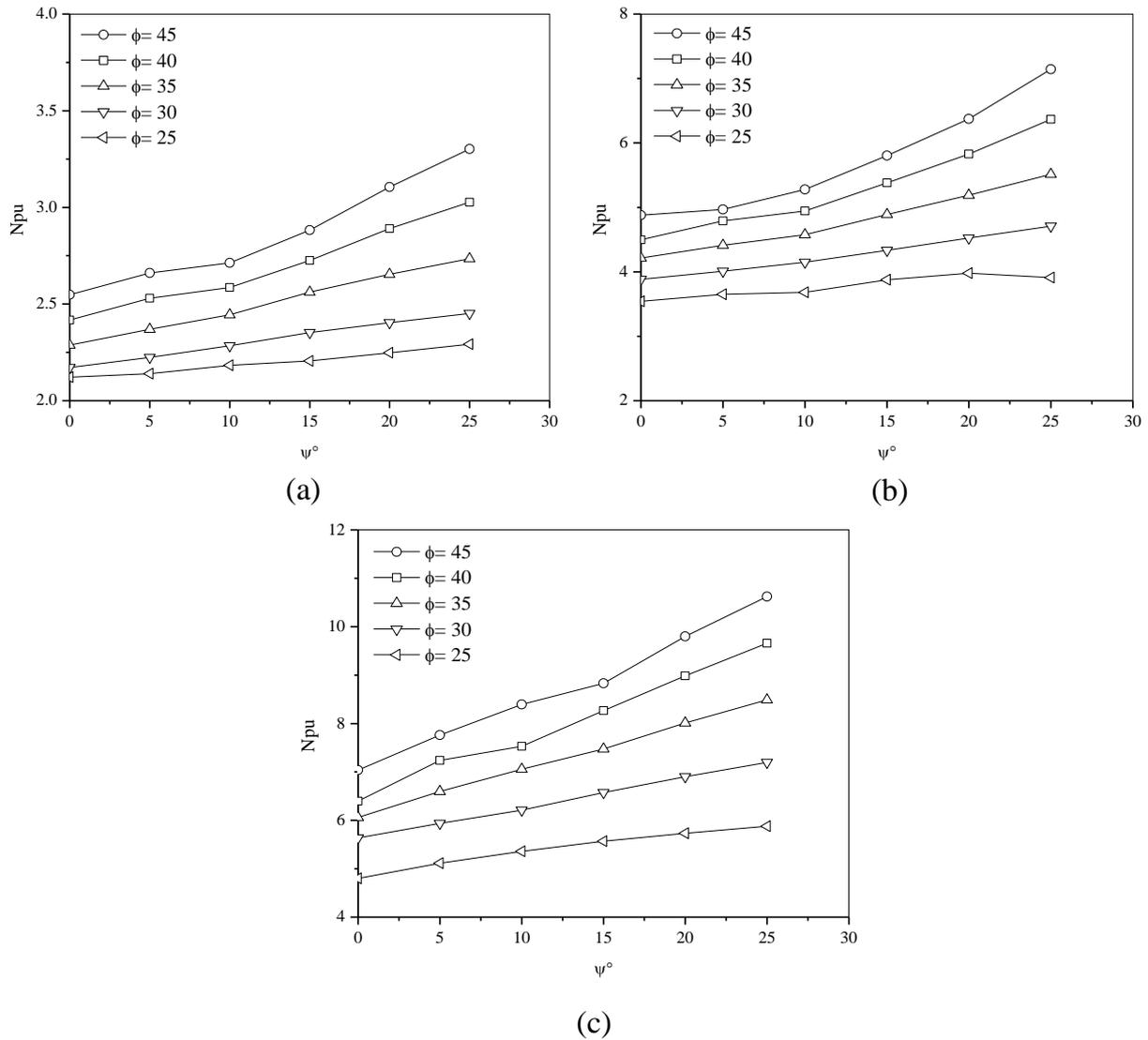


Figure 5: Breakout factors for various H/B and sand properties: (a) H/B=1, (b) H/B=2, and (c) H/B=3

The increases in uplift capacity due to the effect of dilatancy are approximately linear for various embedment ratio and friction angles. Therefore, the breakout factor  $N_{pu}$  for a certain value of dilation angle may be determined conveniently by linearly interpolating between the values of  $N_{pu}$  for the cases that  $\psi=0^0$  and  $25^0$ . The interpolation results are lower than numerical results and the differences are less than about 10%, which is safe for practical engineering. For the convenience of application, the breakout factors are presented in Figure 6, when  $\psi=0^0$  and  $25^0$  for various friction angles and embedment ratios.

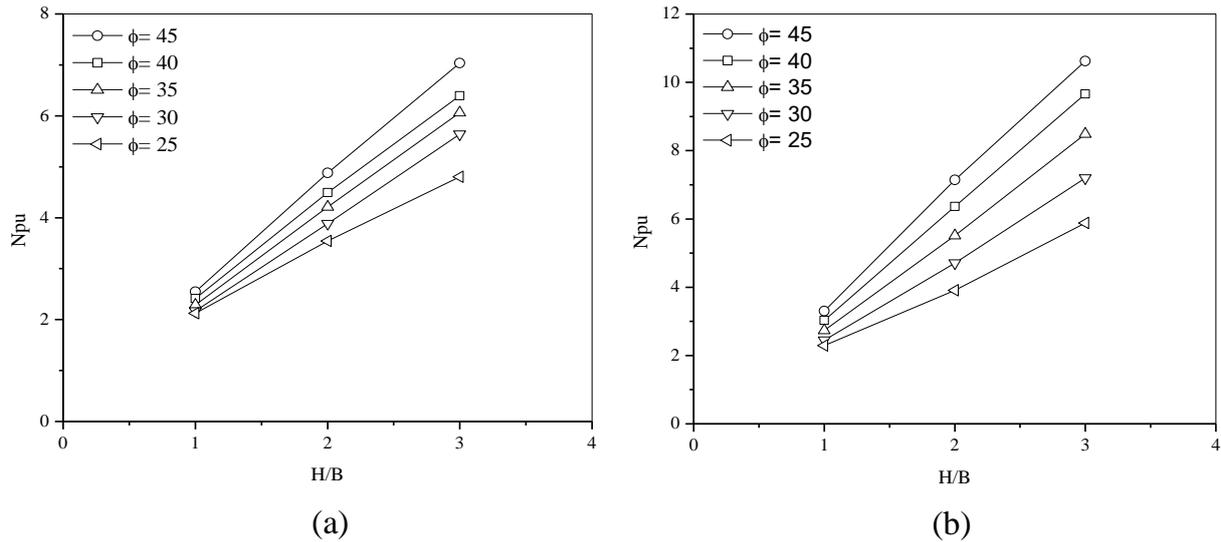
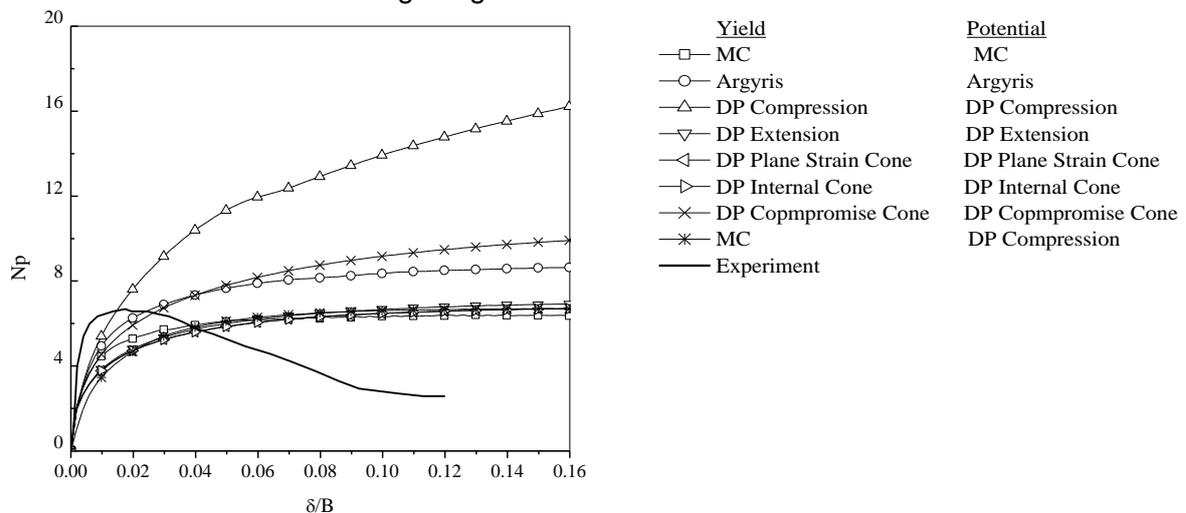


Figure 6: Relationships of H/B and  $N_{pu}$  for various  $\phi$ : (a)  $\psi = 0^\circ$ , and (b)  $\psi = 25^\circ$

### 3.3 Drucker Prager Approximation

The material parameters used for the analysis is shown in Table 1. In the M–C criterion, the predicted failure strength is independent of the intermediate principal stress  $\sigma_2$ , which disagrees with the fact that the biaxial compressive strength is always higher than the uniaxial compressive strength for geomaterials. However, the D–P criterion takes into account the influence of  $\sigma_2$ . From all the curves of Figure 7, it is evident that, Mohr-Coulomb model predicts lower strength at greater strains at all stages of loading compares to Drucker-Prager model. The uplift resistance predicted by the DP compromise cone model is in agreement with the experimental results, though the strength value matches well with the Mohr-Coulomb model predictions. As noted before the strains predicted by Mohr-Coulomb model are conservative. These models do not capture the strain softening behaviour which is found in natural soils according to figure.



(a)  $L/B=1$

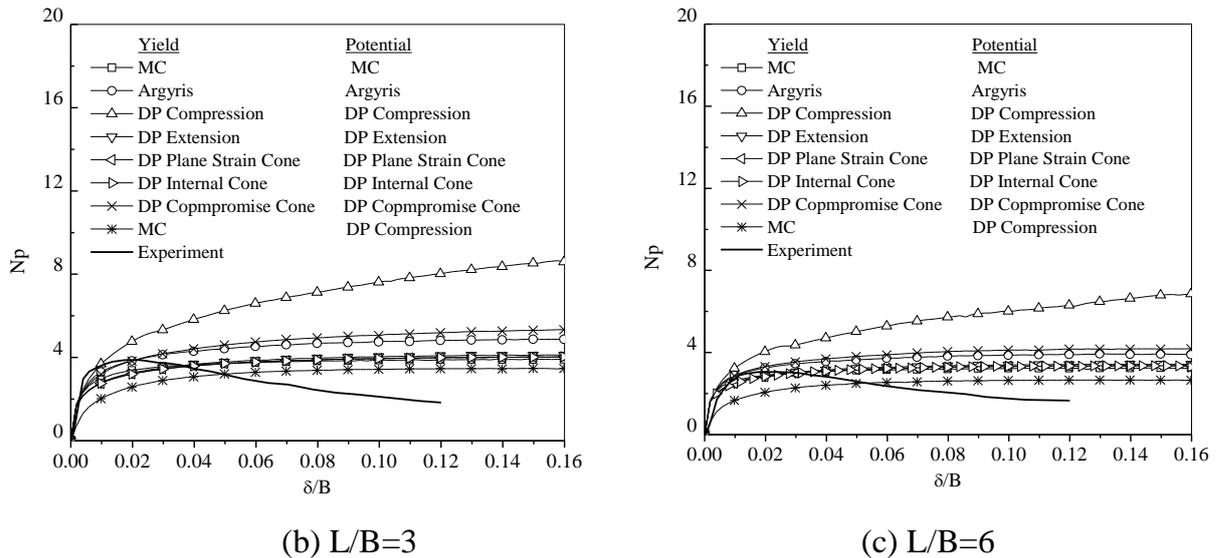


Figure 7: Approximation to Mohr-Coulomb Material Model using 3D FEM

#### 4. CONCLUSIONS

The effects of peak frictional angle, dilatancy, and approximation on the pullout capacity of rectangular anchor foundations buried in dense Toyoura sand is studied through model tests and/or extensive 3D FEM analysis. Dilatancy of sand makes significant effect on the uplift behavior of anchors. The greater dilation angle is resulted from the higher collapse load and displacement when embedment ratio and friction angle are the same. With the increase of friction angle and embedment ratio, the effect of dilatancy is more remarkable. The relationships of dilatancy angles and breakout factors are approximately linear for the same embedment ratio and friction angle. From observations, the Drucker-Prager compromise cone predictions are found to be in better agreement with the experimental results. The Mohr-Coulomb predictions are conservative and hence are in vague in many practical situations in engineering practice. The soils exhibiting post-peak softening, probably due to the presence of natural structure is not predicted by any of these models. Overall, the uplift resistance-displacement factor relationship, response stiffness, and peak uplift resistance factor are the functions of peak frictional angle and dilatancy. Thus, for the practical design and analysis of rectangular anchor foundations, it is necessary to use such a 3D FEM model to handle the issues of soil behavior, geometrical and so on factors.

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