EFFECTS OF WATER-LEVEL VARIATION ON THE STABILITY OF SLOPE BY LEM AND FEM

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ABSTRACT

Stability of waterfront structures such as embankments, dams and natural riverside slopes is hampered due to the variation of water level. Due to the fluid nature, water can easily move into the soil pores. This reduces the effective stress and soil strength that can have devastating effects on the stability of earth slopes. The aim of this paper is to evaluate the effect of the variation of water level on the stability of slope by using limit equilibrium method (LEM) and finite element method (FEM). Fellenius, Bishop, Janbu, Morgenstern-Price and Spencer methods are used for LEM and Mohr Coulomb material model is used for FEM. A numerical model is prepared for LEM and FEM base studies. The geometric boundaries of the model are kept constrained horizontally on the left and right sides and completely fixed at the bottom for FEM based numerical analysis. The geometric model is incorporated in the GEO5, a tool for analyzing both the LEM and FEM based slope stability problems. The material properties are assigned and the numerical analysis is carried out using GEO5. The level of water is varied and the numerical results are computed. From the study, it is noted that the variation of water level affects the factor of safety of slopes. The factor of safety decreases with the increase of the water level. LEM depicts a bit higher factor of safety than FEM.

Keywords: *FEM, factor of safety, LEM, slope stability, water-level*

1. INTRODUCTION

Stability of a slope either man-made or natural is a major concern to the safety of human. Slope stability denotes the resistance of inclined surface to failure. The usual objectives of any slope stability study to the researchers are to find the endangered areas, investigate the potential failure mechanisms, determine of the slope sensitivity to different triggering mechanisms, design the optimal slopes with regard to safety, reliability and economics and design or suggest the possible remedial measures. The factor of safety of such slope is often a function of different parameters such as material (soil) properties, slope geometry, presence of water, earthquake, method of analysis, etc. Due to the importance of the stability analysis of slope from engineering point of view, numerous research works have been reported in the literature (Janbu, 1954; Bishop, 1955; Morgenstern & Price, 1965; He & Zhang, 2012).

Fluctuation of external water level is one of the important factors that influences the stability of waterfront slopes and adjacent land areas. Sources of such fluctuations may include the tidal water-level variations (Ward, 1945; Li, Barry & Pattiaratchi, 1997), variations caused by wind waves (Bakhtyar, Barry, Li, Jeng, & Yeganeh-Bakhtiary, 2009), variations caused by other weather-related events such as heavy rainstorms or snow melting, and combinations of various phenomena (Zhang, 2013). Besides, the natural phenomena and different human activities are also influencing the stability of waterfront slopes. Due to the fluid nature, water can easily move into the soil pores. This reduces the effective stress and soil strength significantly that can have devastating effects on the stability of earth slopes. Hence, the variation of water level is a critical issue that can alter the factor of safety of slope. Conventionally, the limit equilibrium method (LEM) is used to calculate the factor of safety of slope (Cala & Flisiak, 2003; Lin & Cao, 2012). However, this method needs many pre-assumptions. Moreover, only simple models can be analysed by the LEM. With the advanced computational power, the numerical approach such as the finite element method (FEM) has been implemented in the stability analysis of slope (Morgenstern & Price, 1965; Griffiths & Lane, 1999). The main advantage of FEM is that it does not require any prior assumption, even it does not require to assume any predetermined trial failure surface. Moreover, complex shapes of slopes can be easily incorporated.

In this paper, the effect of the variation of water level on the stability of slope has been evaluated. For this reason, a numerical model is prepared and the properties of soil are assigned. Bishop, Spencer, Fellenious, Janbu and Morgenstern-Price methods are used for LEM and Mohr Coulomb material model is used for FEM. In LEM, optimization tool is used to define slip surface of circular shape and in FEM, 6-node triangle element with mesh size of 0.4 m is used to get optimum factor of safety using the software GEO5 (2015).

2. LIMIT EUILIBRIUM METHODS

Several limit equilibrium methods have been developed for slope stability analysis. Fellenius (1936) introduced the first method which is referred to as the Ordinary or the Swedish method for a circular slip surface. Bishop (1955) advanced the first method by introducing a new relationship for the base normal force. The equations for the factor of safety are non-linear. Dividing a potential mass into several vertical slices, Janbu (1954) developed a simplified method for non-circular failure surfaces. Later, Morgenstern-Price (1965), Spencer (1967) and several other researchers made future contributions with different assumptions for the inter-slice forces. All limit equilibrium methods are based on the certain assumptions for the inter-slice normal and shear forces. The basic difference among the limit equilibrium methods lies in how these forces are determined or assumed. Moreover, the shape of the assumed slip surface and the equilibrium conditions for the calculation of the factor of safety are also some points of difference in different limit equilibrium methods. A brief overview of the Fellenius (1936), Janbu (1954), Bishop (1955), Morgenstern-Price (1965) and Spencer (1967) method are outlined in the following sections.

2.1 Fellenius Method

The simplest method of slices assumes only the overall moment equation of equilibrium written with respect to the center of the slip surface. The shear and normal forces between blocks are neglected. The factor of safety is calculated directly from the following expression:

$$FS = \frac{1}{\sum \sin\alpha} \sum[cl + 9N - ul) \tan\varphi] \quad (1)$$

where, u = pore pressure within the block; c, φ= effective values of soil parameters (cohesion and angle of internal friction); W =block weight; N =normal force on the segment of the slip surface; α=inclination of the segment of the slip surface; l = length of the segment of the slip surface

2.2 Janbu Method

Janbu method assumes non-zero forces between blocks. This method satisfies the force equations of equilibrium in the horizontal and vertical directions for all blocks and the moment equation of equilibrium for all but the last slice. In this method, the assumtion is based on the position of forces acting between the blocks. The factor of safety is obtained through the iteration of forces acting between blocks. The inclinations of these forces are then calculated.

2.3 Bishop Method

Bishop simplified method is very common in practice for circular shear surface. This method considers the inter-slice normal forces but neglects the inter-slice shear forces. It further satisfies the vertical force equilibrium to determine the effective base normal force (N), which is given by

$$N = {}^{1}_{m_\alpha} \sum \left(W - \frac{cl \sin\alpha}{F} - ul \cos\alpha \right) \quad (2)$$

where, $$m_a = \left(1 + \tan\alpha \frac{\tan\varphi}{F} \right) \quad (3)$$

2.4 Morgenstern-Price Method

The Morgenstern- Price method also satisfies both the force and moment equilibriums and assumes the inter-slice force function. According to Morgenstern- Price method (1965), the inter-slice force inclination can vary with an arbitrary function $f(x)$ as follows:

$$T = f(x)\lambda E \quad (4)$$

where, $f(x)$ is the inter-slice force function that varies continuously along the slip surface, λ is the scale factor of the assumed function. The method suggests assuming any type of force function, for example, half-sine, trapezoidal or user defined. For a given force function, the interslice forces are computed by iteration procedure. In equations (5) and (6), F_f is equal to F_m which can be given by

$$F_f = \frac{\sum \left[\{ cl + (N - ul) \tan \varphi \} \sec \alpha \right]}{\sum \{ W - (T_2 - T_1) \} \tan \alpha + \sum (E_2 - E_1)} \tag{5}$$

$$F_m = \frac{\sum \{ cl + (N - ul) \tan \varphi \}}{\sum W \sin \alpha} \tag{6}$$

Here, N is the base normal force and E, T are the inter-slice forces.

2.5 Spencer Method

Spencer method is the same as Morgenstern- Price method except the assumption made for inter-slice forces. A constant inclination is assumed for inter-slice forces. The facor of safety is computed for both equilibriums (Spencer 1967). According to this method, the inter-slice shear force is related to:

$$T = E \tan \vartheta \tag{7}$$

3. FINITE ELEMENT METHOD (FEM)

The finite element method is a numerical technique for finding approximate solutions to the boundary value problems. In the finite element method, the actual continuum or body of a matter is divided into smaller and a regular subdivision knows as finite elements. These elements are considered to be interconnected at specified joints called nodes. The nodes lay on the element boundaries where adjacent elements are considered to be connected. In this paper, Mohr-Coulomb material model is used in the finite element method.

3.1 Mohr-Coulomb Model

In FEM analysis using, Mohr-Coulomb material model requires parameters such as angle of internal friction and cohesion, modulus of elasticity, poison’s ratio. The first two parameters serve to define the yield condition. The angle of dilation must also be specified. The failure surface of Mohr-Coulomb model can be expressed as follows:

$$\tau = \sigma \tan \phi + c \tag{8}$$

where, τ represents the shear stress, σ represents the normal stress, ϕ represents the angle of internal friction (slope of the failure envelope) and c represents the cohesion (the intercept of the failure envelope with the τ axis). The Mohr-Coulomb yield surface is represented as a non-uniform hexagonal cone in the principal stress space.

4. GEOMETRY OF THE MODEL

The geometry of the model (slope) without water and with water is depicted in Figures 1 and 2, respectively. The same models with FEM meshes considering the water and without considering the water is depicted in Figures 3 and 4, respectively. A slope with angle of 45 degrees relative to the horizon is considered and the slope height is equal to 6 meters. All the dimensions of the model slope are in meter. Since the objective of this study is to investigate the effect of the variation of water level on the stability of slope, therefore, the stability of the slope without the presence of water is studied first and then, water is considered. The factor of safety of slope both considering the water and without considering the water is computed for the same shape of model by LEM and FEM. Constant phreatic line of water with 5 degree downward inclination is considered with the water level in all cases of analysis. The properties of the material (soil) considered in the present study is shown in Table 1.

Table 1: Material (soil) properties used in the present study

Soil parameters	Value
Cohesion, c (kN/m²)	10
Frictional angel, ϕ (°)	20
Unit weight, γ (kN/m³)	18
Modulus of elasticity, E (MN/m²)	8
Poison's ratio, ν	0.3
Dilation angle, ψ (°)	0.0

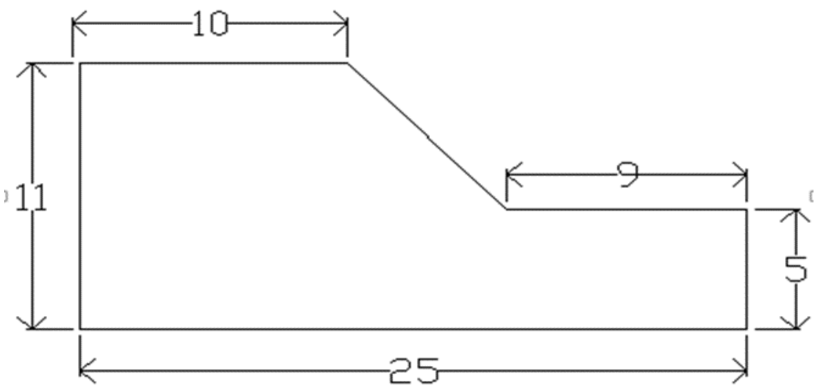

Figure 1: Geometry of the model slope without water considered in the present study

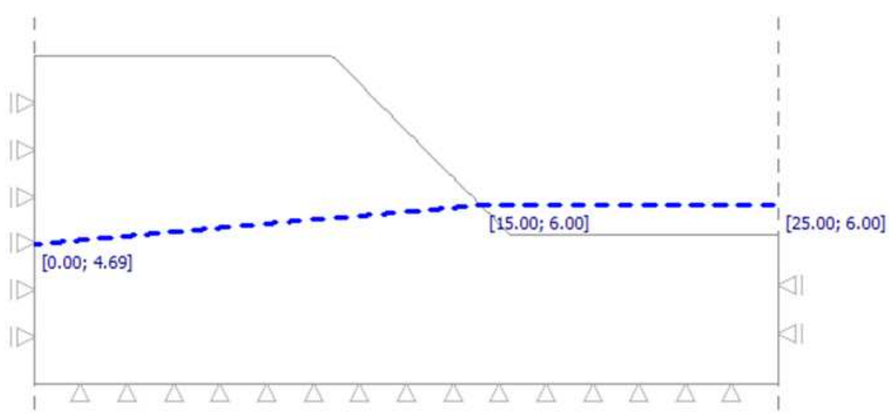

Figure 2: Geometry of the model slope with water considered in the present study

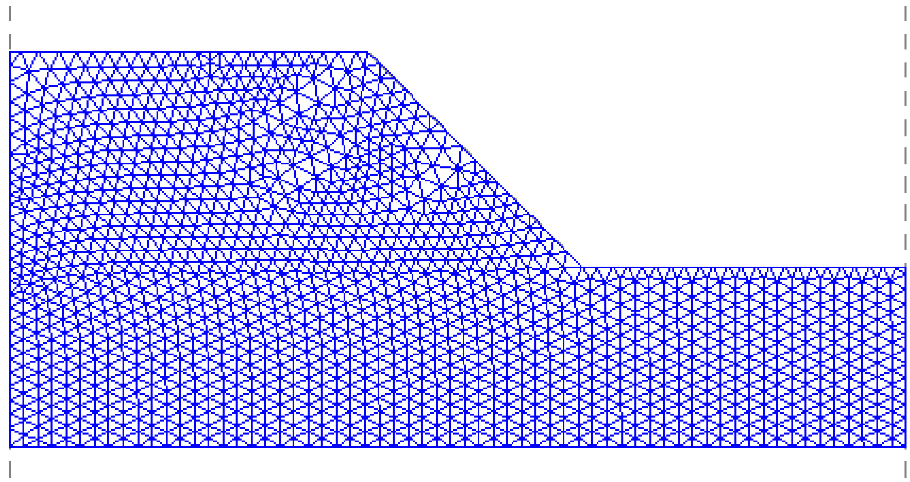

Figure 3: Mesh geometry of the model slope without water considered in the present study

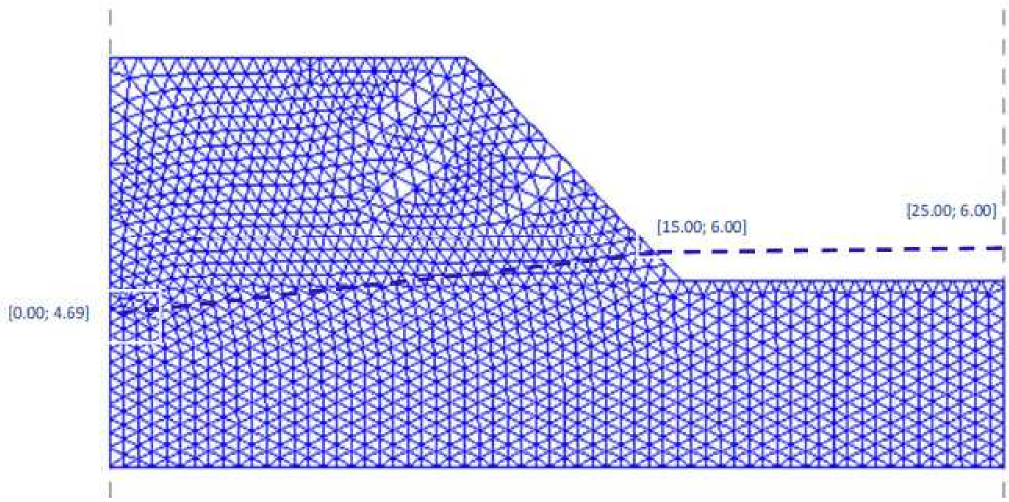

Figure 4: Mesh geometry of the model slope with water considered in the present study

5. RESULTS AND DISCUSSION

A comparison of the LEM and FEM analysis without considering the water is depicted in Table 2. Note that the factor of safety obtained by using Fellenius method yields a bit conservative result compared to the other LEM methods. Note also that the factor of safety by LEM (except Fellenius method) gives higher value than that of Mohr-Coulomb method by FEM when water is not considered. However, it can be stated that the results by LEM and FEM are comparable. A similar comparison is shown in Table 3 for a certain water level. Note that, the pattern is similar except that the factor of safety by LEM is larger than FEM.

The effect of the variation of the water level on the stability of slope in terms of the factor of safety is depicted in Figure 5. It should be noted that the factor of safety, in general, decreases with the increase in the height of water level from the bottom of slope regardless of the methods used. The decrease in the factor of safety is dominant in case of FEM as noted in the Figure 5 for Mohr-Coulomb method. The factor of safety by using Fellenius method yields the lowest value among the LEMs. A reduction in the effective pressure due to the increase in water level contributes to the reduction of the shear strength resulting in the reduction of the factor of safety.

Table 2: A comparison between the LEM and FEM results without considering water

Method of Analysis		Factor of Safety
LEM	Fellenius,	1.18
	Bishop,	1.22
	Janbu,	1.22
	Morgenstern-Price	1.23
	Spencer	1.22
FEM	Mohr-Coulomb	1.20

Table 3: A comparison between the LEM and FEM results with considering water (water level=1.5 m from bottom of slope)

Method of Analysis		Factor of Safety
LEM	Fellenius,	1.16
	Bishop,	1.19
	Janbu,	1.19
	Morgenstern-Price	1.20
	Spencer	1.19
FEM	Mohr-Coulomb	1.13

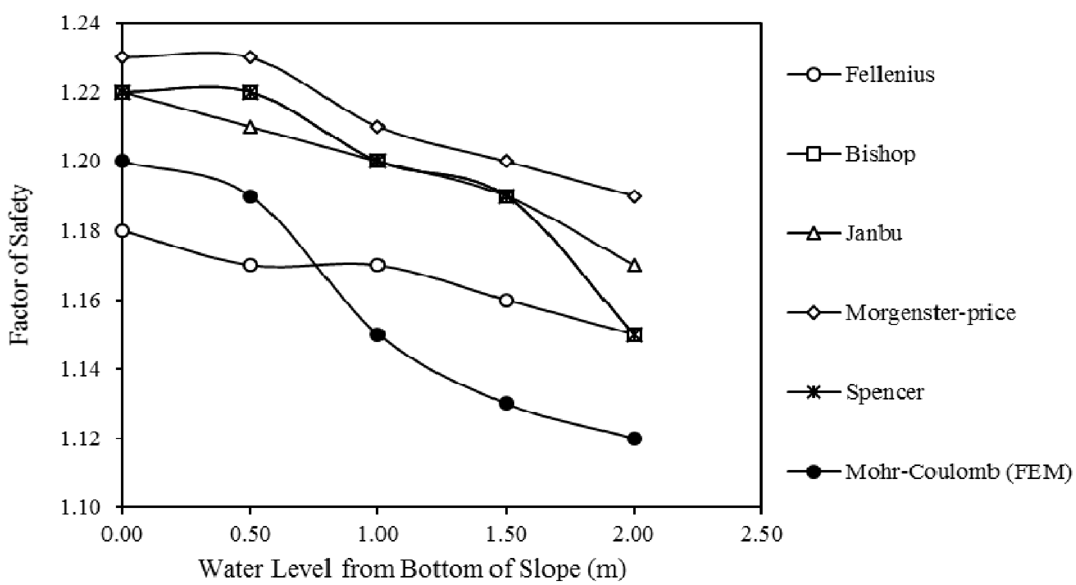

Figure 5: Effect of the variation of water level on the factor of safety by LEM and FEM

6. CONCLUSIONS

A numerical study is carried out to investigate the effect of the variation of the water level on the stability of slope by using LEM and FEM. Fellenius, Bishop, Janbu, Morgenstern-Price and Spencer method are used as LEM and Mohr-Coulomb model is used in FEM. Same model slope is used for LEM and FEM base studies. The geometric boundaries of the model slope are kept constrained horizontally on the left and right sides and completely fixed at the bottom for FEM analysis. The model of slope is incorporated in GEO5 (2015). The material properties are assigned and numerical analysis is carried out using GEO5 (2015). Some of the important findings of the study are summarized as follows:

i. The stability of slope decreases with the increase of water level regardless of the methods used in the slope stability analysis.
ii. The reduction of the factor of safety is dominant in case of FEM than LEM when the height of the water level increases.
iii. LEM yields a bit higher factor of safety than FEM.

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